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Accurate positioning of multiple rocket debris

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Abstract

This paper aims to accurately determine the position and time of sonic explosions in the air by analyzing data from single and multiple debris cases. It focuses on identifying the minimum number of monitoring devices needed, considering random recording time errors, and correcting models to improve positioning accuracy. Additionally, it explores precise positioning strategies when time errors can't be effectively reduced, enhancing airspace debris management. Problem 1: Understanding sound wave propagation, we used a three-ball positioning method. Since both the detection point and the sonic explosion debris are high in the air, the fourth point was used for accuracy. Using least squares method with randomly selected data, we determined the position (longitude: 110.4989, latitude: 27.3105, height: 857.4884m) and time (19.3128s). Problem 2: Theoretically, 44 devices are needed, but four can classify unknown time data groups. Calculations are based on extending the first problem's theoretical basis, introducing relative time and position error solutions. Problem 3: Using actual data, four devices calculate correct locations, but time uncertainty causes larger errors. Introducing additional equipment for secondary screening or multiple data exclusion improves accuracy. Problem 4: Random perturbations of time data (ranging from -0.5 to 0.5) tested the model's anti-interference capabilities. Adjustments included adding an error function and repeating data perturbations until the solution met error range criteria. This significantly reduced time and position errors, with position errors within 1km. This study enhances the efficiency and safety of airspace debris management by improving the accuracy of sonic explosion positioning.

Keywords: rocket debris positioning, four-sphere positioning model, time series

analysis, least squares, random error correction

1|Background

With the continuous progress of space exploration technology, the frequency of rocket use is also increasing. After the rocket is launched, the abandoned stages and boosters of the multi-stage rocket will separate from the main body and eventually fall to the ground. These rocket debris often produce transsonic explosions during falls, causing environmental noise and potential ground damage. In order to reduce these impacts and achieve rapid and safe recovery of debris, accurate positioning of rocket debris becomes crucial. Moreover, accurate positioning can not only facilitate rapid recovery, but also help analyze the causes of rocket failure and improve future designs. At present, it is a common method to receive sonic blast waves through ground monitoring equipment. This process involves complex physical and mathematical modeling, and requires accurate calculation of the propagation time, propagation path, and the distribution of ground equipment. Therefore, building an efficient mathematical model to achieve accurate positioning of multiple rocket debris is a major challenge in the field of space recovery.

In modern space engineering, effective management and recovery of rocket debris is an important link to improve resource utilization and reduce environmental impact. After the multi-stage rocket completes its launch mission, the booster and other components will be separated from the main rocket body through an inter-stage separation device and eventually fall to the ground. These falling debris will produce sonic explosions across the atmosphere, and the detection and analysis of the sound explosion is crucial to the positioning of the debris. In order to accurately determine the landing point of these high-speed moving debris, we need to use mathematical models to simulate the sonic explosion and falling location of each debris to solve the following problems:

Question 1: Build a mathematical model to analyze under what conditions (equipment number and layout), you can accurately determine the exact location (including longitude, latitude and altitude) and time when a single rocket wreck explodes in the air. Considering the limited number of monitoring equipment that may be deployed in practical applications, how can the limited data be used to achieve precise positioning?

Question 2: In practice, a launch mission can involve multiple debris (main rocket stages and multiple boosters), each falling with a sound blast falling. Monitoring equipment may receive multiple sets of vibration waves from different debris simultaneously. Build a mathematical model to analyze this data, determine the specific debris for each set of vibration wave data, and explore at least how much monitoring equipment should be deployed to achieve this goal?

Question 3: Based on the above model, analyze how to extract information from multiple sonic burst data to determine the location of each debris and the time of sonic burst. In addition, it analyzes how to optimize the layout and use of monitoring equipment under various different conditions (such as the number of debris and the time difference of sonic explosive events).

Question 4: How to modify the existing model to more precisely determine the location and time of the debris. Moreover, a strategy is proposed to reduce the effect of time error on localization accuracy to achieve more accurate wreckage positioning.

2|Problem analysis

2.1|Analysis of Problem 1

When analyzing the accurate positioning problem of multiple rocket debris, it is first assumed that the signal propagation process is not disturbed by any dynamic environmental changes, such as moving obstacles, changes of multipath effect, attenuation or multipath effect. Based on this assumption, considering the sound wave propagating along a gradually expanding sphere in the air, with the detection device that receives the sonic explosion centered, the common point where the sphere intersects is the location of the sonic explosion. Usually, the traditional three-sphere localization theory[1]Applicable to satellites in the sky to locate points on the ground, that is, a ground point from the intersection of three known sphere (such as the sphere). However, our requirement is to calculate the points in the air, and the debris is in the threshold space of one point that does not actually exclude the other point, so the fourth ball is used to correct the positioning task accurately with only three balls. Why do you choose the four detection sites specifically? When there are only three detection points, unless these three spheres are tangent at one point, their intersection solutions are usually not unique, affecting the accuracy of the practical application. Therefore, using detection points that are not coplanar in the four spaces improves the uniqueness and accuracy of localization.

In the preliminary calculation, the height of 130,000 meters and the time of-800 seconds calculated with seven devices is obviously not practical, suggesting that there may be errors in the data. Four probe points were randomly selected to recombine and model building in the relative coordinate system. The results showed significantly inconsistent positions in the scatter plot of possible positions drawn by point A (0,0,824). Further data evaluation revealed abnormal data for the D and F devices, which may result from accuracy error, ambient noise, or device clock synchronization problems. Therefore, it was decided to exclude data from both devices and recalrecalto obtain more accurate estimates of the sonic burst location.

In the modified calculation, the use of the sphere relative coordinate system is abandoned, and the distance between latitude and longitude difference multiplied by the conversion coefficient of the corresponding earth surface distance is directly used to calculate the distance, and ensure that all calculations are positive. The corrected time is calculated as

$$t_i = t_0 + \frac{d_i}{c} \tag{1}$$

Negative number errors in the directional sense are avoided. Through these steps, several possible results are obtained, through theoretical calculation and cross-over de-verifying the correctness of the data processing and model building methods.

2.1|Analysis of Problem 2

The second problem uses the spatial geometry relationship to determine the source debris of the vibration wave received by the monitoring equipment. The positioning error and accuracy of the multipoint positioning system are closely related to the number of ground equipment and the solution algorithm of location points[2]. The monitoring equipment is used as a point in the three-dimensional sphere coordinate system, with longitude, latitude and elevation information. Set four debris and multiple monitoring devices, each receiving multiple sets of vibration waves. By comparing the time order of the arrival of vibration waves, a part of the space area is preliminarily excluded, and the equipment is selected as the reference point to determine the possible explosion points, gradually eliminate the illogical position combined with the data of other equipment, and finally the determination of the position and time of the explosion point. Time-series data ordering and multi-source data analysis are introduced to handle the error through the squared error function to enhance the sensitivity to small deviations. Use two kinds

error function:

$$\Delta W_1 = \left(t_i - \frac{\sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2}}{340} - t_0 \right)^2 \tag{2}$$

A calculation of the difference between the distance from the explosion source and the theoretical propagation time,

$$\Delta W_2 = \left(\sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2} - 340(t_i - t_0) \right)^2 \tag{3}$$

Another focus on the distance error and time error square, verify the data consistency and accuracy, to ensure that the solution to adapt to the actual application scenario, theoretically need 16 monitoring group (4x4) to independently track four debris, but for the model, after fusion data, a device can monitor multiple debris error function narrow matching, that is, in practice, only four monitoring equipment, because each device can receive multiple sets of data from different debris, and through the part has discussed the error function optimization algorithm accurately distinguish and match to their own debris.

2.1|Analysis of Problem 3

According to the model in question 2, the given monitoring equipment data is used to determine the location and time of the sonic explosion of the four debris in the air. The arrival time of each device is combined with the 3 D coordinates of the device to calculate the theoretical source position of each sonic explosion. By conducting permutation and combination analysis of the time data on the basis of four sets of vibration waves received by each device, we determine which group of time data belongs to the same sonic blast event, so as to correctly match each sonic blast with its source debris.

In the initial stage, a monitoring device (e. g. device A) is selected as a reference to first match the first set of time data recorded by the device. After success, the data are removed and if the next set of time is not matched, continue to try subsequent data. This process ensures that each set of time data is checked one by one.

After completing each round of matching, the relevant data were removed and the remaining data were further accurately matched. This step improves the positioning accuracy by minimizing the matching error and continues until all time data find the appropriate match, thus accurately determining the location and explosion time of each remnant.

2.1|Analysis of Problem 4

Aiming at question 4, in this study, question 4 focuses on how to handle and optimize the positioning accuracy of the rocket debris in the presence of random error in the sonic burst time recording. We propose a method that simulates uncertainty in realistic situations by introducing stochastic time error and performs error correction using mathematical models to minimize the difference between observation and prediction times. The model relies on repeated iterations and optimization, combining a small number of key data points and verifying it through other data points to ensure the accuracy of the solution. Moreover, the antiresistance and stability of the model were further verified by multiple disturbance experiments to achieve the goal of reliably locating debris in practical applications.

3|Hypothesis

1. Assuming that the environment is static, and the signal interference caused by moving obstacles, multipath effect change, attenuation or multipath effect are not considered.
2. It is assumed that the errors in the data mainly come from the insufficient accuracy of the detection equipment, environmental noise or clock synchronization problems.
3. Suppose that the vibration wave spreads along the gradually expanding sphere, with no other medium interference in the middle, and the propagation speed is fixed, which is 340 m / s.
4. 4 Suppose that the curvature of the ground can be ignored when calculating the distance between two points.
5. Suppose that the distance per degree between longitude is approximately 97.304km, while the distance between latitude is approximately 111.263km.
6. Suppose that the site of a single-tone burst is a moment point, and there is no site shift.

4|Symbol Description

Symbol	Meaning
di	Straight-line distance from the i th detection device to the sound burst birth point
(x0,y0,z0)	The spatial coordinates of the sound burst
(xi,yi,zi)	The spatial coordinates of the i th probe device
t0	The time of the sound burst
ti	The time that the sonic burst signal reaches the i th detection device
c	Speed of sound, used to convert the spatial distance into a time difference
P	Soundburst current position
Psourse	Sound explosion source position
Δt	time error
Δd	range error

5|Model building and solution

5.1|Analysis and solution of Problem 1

5.1.1|Thinking of Problem 1

The third part "assumptions of the model" allows us to reduce the problem to purely geometric and physical problems, thus focusing on the problem of signal propagation and reception. In the model, we know that sound waves propagate in the air along a gradually expanding sphere. This means that a sphere is centered around any detection device that receives the sonic burst and the distance of the sound wave travels as the radius. The common point where multiple such spheres intersect is the place where the sonic explosion occurs.

The traditional three-ball positioning theory applies to points on the ground to locate satellites in the sky, which determines a ground point from the intersection of three spheres of known positions (such as the sphere). However, our requirement is to calculate the points in the air, and the debris is in the threshold space of one point that does not actually exclude the other point, so the fourth ball is used to correct the positioning task accurately with only three balls. Why choose the four detection points? In the case of only three probe points, unless any two of them are tangent at exactly one point, which is difficult to obtain accurate localization results in practice. Therefore, using detection points that are not coplanar in the four spaces improves the uniqueness and accuracy of localization. For the algorithm of multipoint positioning system, it can be roughly divided into two algorithms, one is the algorithm of arrival time, the other is the calculation of arrival time difference. In many algorithms, the time difference positioning is widely used. It can overcome the ranging error caused by the clock asynchronization between the ground equipment and the target, so as to improve the positioning accuracy[3].Our localization model is based on the following equation:

$$d_i^2 = (x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2 \tag{4}$$

$$t_0 - t_i = \frac{d_i}{c} \tag{5}$$

Using the time of the sonic burst signal received by each device, combined with the three-dimensional sphere coordinates of the device and the known speed of sound, establishes the Eq. These equations will relate the position and time of bursts. By solving this set of equations, we can accurately determine the position and time of bursts, as follows:

$$\begin{cases} (x_0 - x_A)^2 + (y_0 - y_A)^2 + (z_0 - z_A)^2 = d_A^2 \\ (x_0 - x_B)^2 + (y_0 - y_B)^2 + (z_0 - z_B)^2 = d_B^2 \\ (x_0 - x_C)^2 + (y_0 - y_C)^2 + (z_0 - z_C)^2 = d_C^2 \\ (x_0 - x_D)^2 + (y_0 - y_D)^2 + (z_0 - z_D)^2 = d_D^2 \end{cases} \tag{6}$$

$$\begin{cases} t_0 = t_A - \dfrac{d_A}{c} \\ t_0 = t_B - \dfrac{d_B}{c} \\ t_0 = t_C - \dfrac{d_C}{c} \\ t_0 = t_D - \dfrac{d_D}{c} \end{cases} \tag{7}$$

5.1.2|Data pre-processing

First, seven devices are used to calculate together to obtain an unrealistic value with A height of 130,000 meters and A time of-800 seconds. Now, four detection points are randomly selected to establish the relative coordinate system. Assuming one origin, A has (0,0,824) relative to the origin, as shown in Table 1 and 2:

Table 1. The 3 d coordinates of each equipment and the arrival time of the sonic explosion

equipment	longitude (°)	latitude (°)	altitude (m)	Sonic blast arrival time
A	110.241	27.204	824	100.767
B	110.780	27.456	727	112.220
C	110.712	27.785	742	188.020
D	110.251	27.825	850	258.985
E	110.524	27.617	786	118.443
F	110.467	27.921	678	266.871
G	110.047	27.121	575	163.024

Table 2. Three-dimensional coordinates of each device and the arrival time of the sound blast relative to point A

Equipment	Difference in longitude	Difference in latitude	Altitude
A	0	0	824
B	0.539	0.252	727
C	0.471	0.581	742
D	0.010	0.621	850
E	0.283	0.413	786
F	0.226	0.717	678
G	-0.194	-0.083	575
Equipment	**Longitude difference to meter**	**Latiinal to meter**	**(X, Y, Z)**
A	0	0	(0, 0, 824)
B	52448	28042	(52448, 28042, 727)
C	45829	64634	(45829, 64634, 742)
D	973	69106	(973, 69106, 850)
E	27528	45954	(27528, 45954, 786)
F	21987	79786	(21987, 79786, 678)
G	-18879	-9240	(-18879, -9240, 575)

Point A is set as the origin in the horizontal direction and 824 meters in the vertical direction (usually the height), drawing a scatter plot of possible positions at point A (0,0,824), as shown in Figure 5.1 and 5.2:

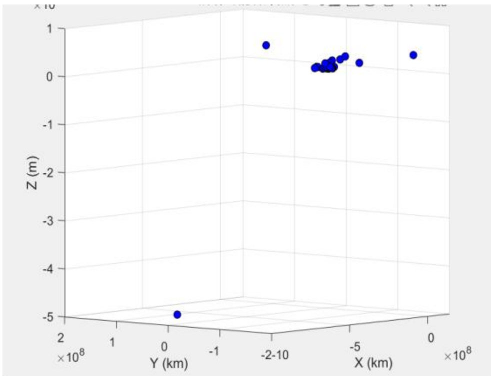

Figure 5.1 Location of all possible sonic burst points after excluding A

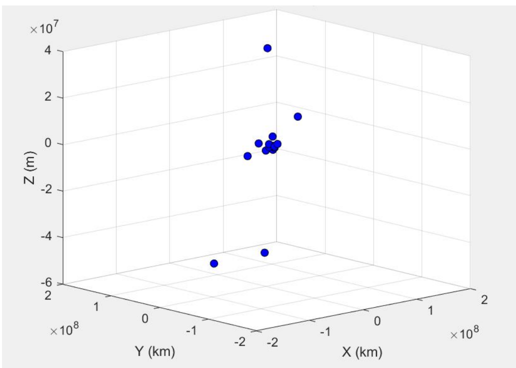

Figure 5.2 Scatterplots of all possible sonic burst locations

In Figure 5.1 to Figure 5.8, each scatter represents the inferred location of possible sound bursts based on data collected by multiple detection devices. These positions were obtained by calculating the distance of each detection device to the sonic burst source. It is obvious from the figure that the scatter distribution is not uniform and some positions significantly deviate from the main group, suggesting a possible error in part of the data. To further test this, we randomly removed one data point and redrew the scatter plots for seven different scenarios to observe the change.

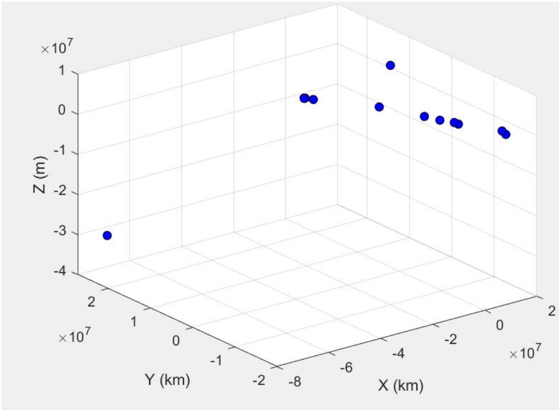

Figure 5.3 Location of all possible sonic burst points after excluding B

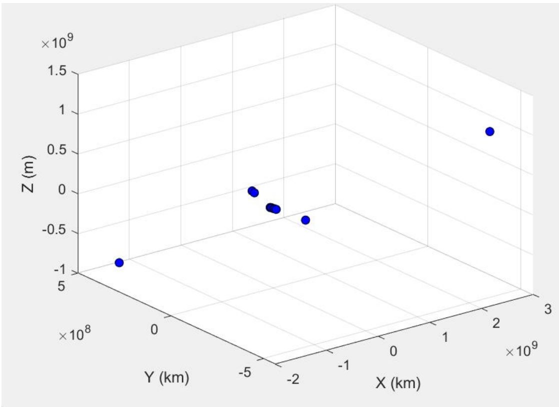

Figure 5.4 Location of all possible sonic burst points after excluding C

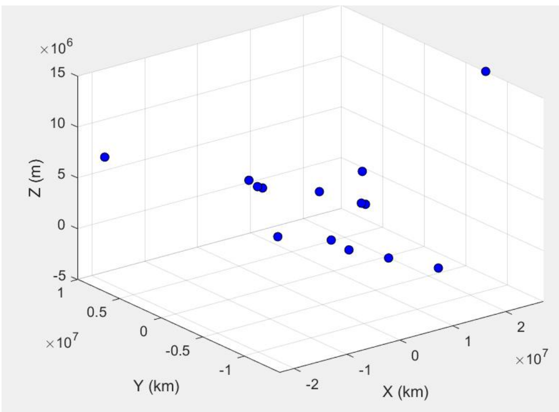

Figure 5.5 Location of all possible sonic burst points after excluding D

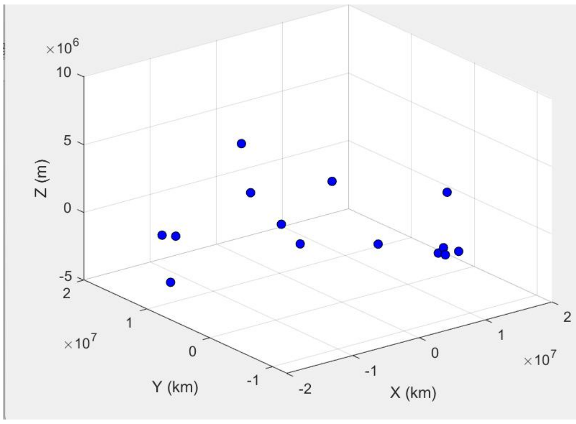

Figure 5.6 Location of all possible sonic burst points after excluding E

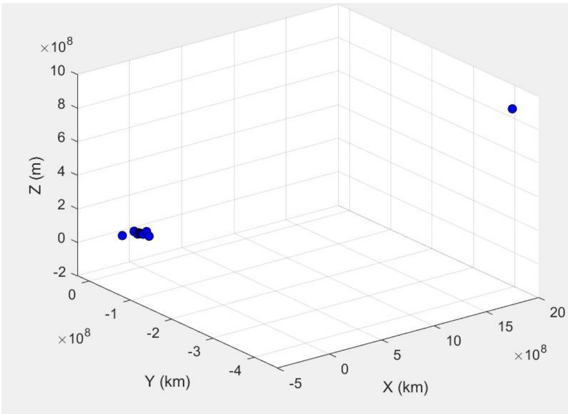

Figure 5.7 Location of all possible sonic burst points after excluding F

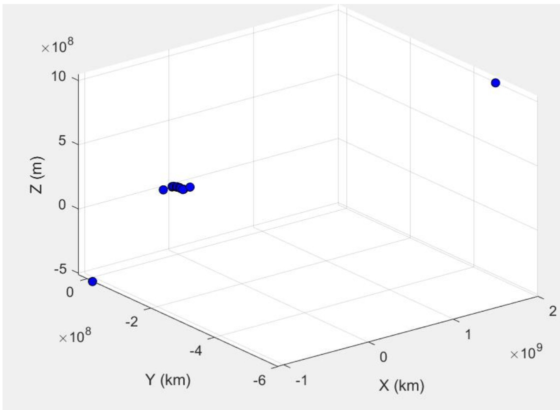

Figure 5.8 Location of all possible sonic burst points after removing G

After detailed analysis, there are indeed data errors rather than arithmetic problems. After a comprehensive evaluation of the data, it is obvious that the data of D and F devices are abnormal, which may be caused by accuracy error, environmental noise or equipment clock synchronization problems. Therefore, we decided to exclude the data of D and F devices, recalculate, after excluding the data of D and F (x0， y0， z0， t0) The scatter plot is very concentrated.

5.1.3|Model establishment and solution of Problem 1

The goal of this calculation is to minimize the equation while excluding data from the D and F devices.

$$t_i = t_0 + \frac{d_i}{c} \tag{8}$$

In order to obtain a more accurate estimate of the sonic burst location. After recalculation, we obtained several possible results, which is actually based on our modeling thinking in problem 2 (based on each set of monitoring equipment, to determine the possible location of the debris. If we choose devices A and B as a benchmark, then we can take the connection between devices A and B as an axis, and a certain point on this axis is a possible explosion point. Then, we consider the vibration waves received by other devices, gradually eliminate the unqualified space, and finally determine the possible explosion point.) The recalculation results are as follows:

Table 3. Preliminary estimation results of sonic burst locations after excluding D and F

Solution 1	(x0,y0,z0)=（2.702，39.680，819.034）	t0=162.127s
Solution 2	(x0,y0,z0)=（2.416，40.143，825.190）	t0=161.926s
Solution 3	(x0,y0,z0)=（1.412，40.514，823.105）	t0=161.796s
Solution 4	(x0,y0,z0)=（1.771，39.713，822.590）	t0=161.997s
Solution 5	(x0,y0,z0)=（2.292，39.803，820.939）	t0=162.067s

Table 4. Results of the optimized sonic burst location estimation

Solution 1	(x0,y0,z0)=（5.321,41.565,825.510）	t0=156.961s
Solution 2	(x0,y0,z0)=（5.253,41.573,824.530）	t0=156.980s
Solution 3	(x0,y0,z0)=（6.016,41.532,825.973）	t0=156.937s
Solution 4	(x0,y0,z0)=（5.470,41.519,825.001）	t0=156.966s
Solution 5	(x0,y0,z0)=（5.469,41.544,825.448）	t0=156.955s

From Table 3 and Table 4, it can be found that the results are very close to each other, but there is an obvious problem: the sonic burst time is greater than the sonic burst detection time, which obviously does not conform to the physical reality. After inspection found in the calculation of the negative sense of direction, so decided to abandon, using the relative coordinate system (previously used sphere coordinate system instead than cartesian coordinate system), but use the geographic coordinate based method, namely directly using latitude gap distance, longitude gap distance, relative coordinate system is x (km) y (km) z (m), not relative x (m) y (m) z (m).

$$d_i = \sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2} \tag{9}$$

At the same time, the corrected time calculation formula is:

$$t_i = t_0 + \frac{d_i}{c} \tag{10}$$

Take addition and the square root to minimize the following expression:

$$t = \frac{\sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2}}{v} + t_0 - t_i \quad (11)$$

The results of random one of the other A, B, C, E, and G are shown in Table 5, Table 6:

Table 5. Optimized sonic burst position prediction

optimum position (x0,y0,z0): （Longitude (m), latitude (m), elevation (m)）	The best time t0(s)
(x0,y0,z0):(10751985.0262 ,3038649.9163 ,857.4884)	t0: 19.3128
(x0,y0,z0):(10751956.4806 ,3038604.4719 ,802.8973)	t0: 19.1681
(x0,y0,z0):(10752003.4645 ,3038676.7295 ,823.3546)	t0: 19.4009
(x0,y0,z0):(10751882.6159 ,3039069.4269 ,1148.6469)	t0: 19.0941
(x0,y0,z0):(10752719.2369 ,3037498.3461 ,1138.1194)	t0: 18.7906
optimum position (x0,y0,z0): （Longitude (°), latitude (°), elevation (m)）	The best time t0(s)
(x0,y0,z0):(110.4989, 27.3105, 857.4884)	t0: 19.3128
(x0,y0,z0):(110.4986, 27.3101, 802.8973)	t0: 19.1681
(x0,y0,z0):(110.4991, 27.3108, 823.3546)	t0: 19.4009
(x0,y0,z0):(110.4978, 27.3142, 1148.6469)	t0: 19.0941
(x0,y0,z0):(110.5064, 27.3002, 1138.1194)	t0: 18.7906

According to Table 3., the results can be found to be very close together

Table 6. Theoretical optimal location location and time

optimum position (x0,y0,z0): （Longitude (m), latitude (m), elevation (m)）	The best time t0(s)
(x0,y0,z0):(10751985.0262 ,3038649.9163 ,857.4884)	t0: 19.3128
(x0,y0,z0): （Longitude (°), latitude (°), elevation (m)）	t0(s)
(x0,y0,z0):(110.4989， 27.3105， 857.4884)	t0: 19.3128

According to the theoretical calculation of sound burst position and sound burst time:

That is, Figure 5.9 is the result of B, C, E and G results. It is found that D and F have great error. The corrected data is very close to the location and time of sonic burst predicted by the model, proving that the data preprocessing is correct and the result is correct. The calculated position and distance, A, B, C, D, E, F, and G should be the "relative" sonic burst time, respectively.

Table 7. Cross-validation of the sonic burst localization results

The "relative" sonic burst time of the observation point A is:	19.06 s
The "relative" sonic burst time of the observation point B is:	18.83 s
The "relative" sonic burst time at the observation point C is:	21.29 s
The "relative" sonic burst time of the observation point D is:	76.34 s
The "relative" sonic burst time at the observation point E is:	17.97 s
The "relative" sonic burst time of the observation point F is:	66.96 s
The "relative" sonic burst time of the observation point G is:	19.51 s

The results in Table 7 above use the nonlinear least squares method to optimize estimates of position and time. The goal is to minimize the sum of the difference between the prediction and the actual arrival time for the location and time of the explosion. Preliminary estimates are provided (x0, y0, z0, t0) And algorithm options, especially with high iterations and very small step tolerance to ensure the algorithm is as close to the global optimal solution as possible. Using genetic iteration, the optimized location and time of sound explosion (please refer to Appendix 9.1 Matlab for detailed code), which provides high precision spatial and temporal coordinates. See Figure 5.9 (Due to the visual error of the viewing perspective, it is actually consistent with the data):

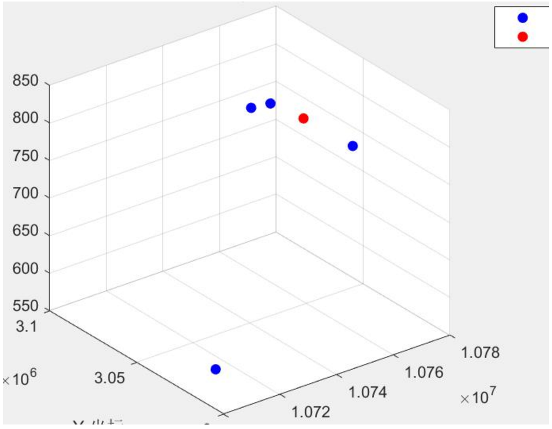

Figure 5.9 3 D positions of observation points and sonic bursts

In Figure 5.9. Blue is the observation point, Red is the sonic boom point, and the diagram below is consistent with this one. In conclusion, the location and time of the debris are longitude (°): 110.4989, latitude (°): 27.3105, elevation (m): 857.4884, time (s): 19.3128.

5.1.4|Model validation of Problem 1

The basic formula we used to calculate the time error is:

$$\Delta t = t_i - \frac{|P_i - P_{\text{sourse}}|}{c} - t_0 \tag{12}$$

The basic formula we used to calculate the distance error is:

$$\Delta d = |d_i - c \cdot (t_i - t_0)| \tag{13}$$

As shown in Table 8, the model accuracy and the model accuracy is to prove that the optimal solution position is correct, and the unknown time is to match (Appendix 9.4 VScode C + +) error result.

Table 8. Model validation error results

Time errors and position errors of A:-0.34090996 s, 115.91016 m
Temporerror and position error of B:-0.57465417 s,195.28125 m
Time errors and position errors of C:1.8924046 s,-643.41797 m
Time errors and position errors of D:56.936317 s,-19358.355 m
Time and position errors of E:-1.4274044 s,485.32031 m
Time errors and position errors of F:47.560127 s,-16170.445 m
Time errors and position errors of G:0.10875893 s,-36.980469 m

Time error: the positive value indicates that the actual arrival time is later than predicted, and the negative value indicates that the actual arrival time is earlier, the closer the absolute value is to 0, the smaller the error; the surface time of the model, the devices D and F show abnormally large positive time error, 56.936317 seconds and 47.560127 seconds respectively, which may indicate that the signals received by the two devices have significant problems or data processing, and the time error of other devices is within our acceptable error range (2 seconds fluctuation). Distance error: the larger the absolute value indicates that the actual measured distance is farther than the model prediction, the closer the absolute value is, the smaller the zero error; the absolute distance error of the equipment D and F is also shown as an unusually large value, 19358.355 m and 160170.445 m respectively, which further proves that the data of these two devices may have serious problems. The errors of other equipment are within our acceptable range (within 1km).

5.1.5|Error Analysis of Problem 1 model

During the data analysis phase of this study, we identified that data recorded by devices C and G also showed slight error. These errors were found by subsequent enhanced statistical analysis and comparative analysis with other device data. Although all possible sources of data error should have ideally be excluded, given the limited number of data points available, we chose to keep these data to maintain a sufficient amount of data to support reliable statistical inference. During data preprocessing, we implemented multiple techniques to reduce the impact of these small errors, including data smoothing and outlier processing. Furthermore, our model design takes these factors into account, ensuring the robustness of the results. Nevertheless, the presence of these small errors remains a limitation of our study. Future work will focus on improving the precision of data acquisition and increasing monitoring sites to further validate our findings.

5.2|Analysis and solution of Problem 2

5.2.1|Thinking of Problem 2

The specific four steps to determine which of the vibration waves received by the monitoring device are shown below:

Step1: We can view the position of the monitoring device as a point in the three-dimensional sphere coordinate system. Each monitoring device can be regarded as a coordinate point with longitude, latitude, and elevation information.

Step2: We assume four debris and multiple monitoring devices. If each monitoring device receives multiple sets of vibration waves in chronological order, we can first exclude a part of the space according to the chronological order that the vibration waves arrive. For example, if device A received the vibration wave before device B, then we can determine that the explosion point is not within the spatial range between devices A and B.

Step3: We can consider using each set of monitoring equipment to determine the possible location of the debris. If we choose devices A and B as a benchmark, then we can take the connection between devices A and B as an axis, and a certain point on this axis is a possible explosion point.

Step4: We will consider the vibration waves received by other devices, gradually eliminate the space that does not meet the conditions, and finally determine the possible explosion point.

Step 5: The least squares method solution, the principle is briefly explained below:

After converting the sphere coordinates to the Cartesian coordinates:

The sphere model with the center of the detector can be described as

$$R^2 = (x_0 - a)^2 + (y_0 - b)^2 + (z_0 - c)^2 \tag{14}$$

In formula _ (1), (a, b, c) are used to describe the coordinates in the coordinate system. Equation (1) can be rewritten as

$$2ax_0 + 2by_0 + 2cz_0 + R^2 - (a^2 + b^2 + c^2) = x_0^2 + y_0^2 + z_0^2 \tag{15}$$

For s points, equation (2) can be described in the form of a matrix, then you have

$$\begin{bmatrix} 2x_0 & 2y_0 & 2z_0 & 1 \\ 2x_1 & 2y_1 & 2z_1 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 2x_s & 2y_s & 2z_s & 1 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ R^2 - (a^2 + b^2 + c^2) \end{bmatrix} = [x_0^2 + y_0^2 + z_0^2, x_1^2 + y_1^2 + z_1^2, \cdots, x_s^2 + y_s^2 + z_s^2] \tag{16}$$

Formula (3) can be abbreviated as

$$A \cdot m = B \tag{17}$$

It is solved by the least squares method, yes

$$m = (A^T \cdot A)^{-1} \cdot A^T \cdot B \tag{18}$$

So as to obtain the coordinates of the center of the rocket debris, to achieve the position of the rocket debris when the sonic explosion.

Specific idea: The three-dimensional coordinates (longitude, latitude and elevation) and four possible arrival time of the monitoring station are input into the model. Set the solver options for the nonlinear least squares method, the maximum number of iterations and tolerance. For the full burst time of each monitoring station, an observation equation was defined for each permutation and then solved using least squares. For each time combination, the sum of time errors and position errors was calculated. The results of each iteration (position, time, error) are stored, and the result with the smallest error is selected as the optimal solution. Output all results satisfying the conditions including z coordinates greater than 0, time and position errors less than the set threshold.

In practice, with each group of different monitoring equipment as the benchmark, repeat the above process, gradually reduce the possible range of the explosion point location, and finally determine the location and time of the explosion point.

This method uses the spatial geometry relationship, and gradually determines the position and time of the explosion point by excluding the space of non-conforming conditions, thus realizing the accurate positioning of multiple debris.

5.2.2|Establish the debris source location model

In this question, it is how to determine which detected vibration wave belongs to which debris. Due to the presence of multiple debris and sound bursts at the same time, each detection device will receive vibration waves from different debris in turn, forming multiple sets of time series data, with random error and multi-source data challenges. To ensure the accuracy and reliability of the data analysis, we adopted the squared error function because the squared operation can avoid the potential analysis bias caused by positive and negative errors offset, thus enhancing the sensitivity of the model to small deviations. How to determine the source of the debris from the vibration waves received by the monitoring device? This question is answered in three steps, as follows:

Step1: First, order the four sets of sonic burst data, select the specific time point in each group of data, and based on the subsequent arrangement and combination.

Step2: After the four rows of time data are arranged and combined, select the data of the first row, and then the last three rows are arranged and combined.

Note: Since there must be four numbers, the start bit does not affect the result bit.

Step3: Subsequently the optimal solution for all permutation combinations (x0,y0,z0,t0) Contrthe two error functions.

The core idea of this approach is that, in the case of multi-source data, by systematically examining all possible data combinations, we can more comprehensively evaluate the various scenarios and ensure that the best solution is found. Specifically to the application of error function, we designed two main error evaluation methods to determine the optimal solution:

The first error function

$$\Delta W_1 = \left(t_i - \frac{\sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2}}{340} - t_0\right)^2 \tag{19}$$

This function primarily evaluates the square of the distance from each monitoring device to the sonic burst source against the theoretical propagation time. This function helps us to evaluate the relationship between temporal error and spatial distance.

The second error function

$$\Delta W_2 = \left(\sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2} - 340(t_i - t_0)\right)^2 \tag{20}$$

Focusing on the direct squared difference between the distance error and the time error, this helps us to verify the magnitude of the error from another perspective. By the comparison of these two error functions, we were able to identify which set of data provides the time-space configuration closest to the actual situation. The advantage of this approach is that it allows us to validate the consistency and accuracy of the data from multiple perspectives, ensuring that our solution can effectively respond to complex practical application scenarios.

5.2.3|Analysis of the minimum number of monitoring equipment required to determine the location of the ideal debris

According to the answer to a direct reasoning, theoretically need 16 monitoring group (4x4) to independently track four debris, but for practical model, after fusion data, a device can monitor multiple debris of error function narrow matching, that is, in practice, only four monitoring equipment, because each equipment can receive multiple sets of data from different debris, and through the previous part has discussed the error function optimization algorithm accurately distinguish and match to their debris, we will prove in the actual calculation of question three. The use of four devices can ideally determine the correct location, but in practice, additional equipment is often needed to ensure accurate screening of locations due to possible data errors and computing power limitations. Here we will prove our point of view: in question 3, only four equipment to successfully determine the correct point. However, our auxiliary validation uses all the seven devices, which is the difference between model requirements and practical application: the model of problem 3 sets specific parameter requirements, such as the square sum of time error less than 1, the square sum of position error less than 40000, and z0 must be greater than 0. These conditions indicate that four devices are sufficient to complete the task only if the data is correct and the computing power is sufficient.

5.3|Analysis and solution of Problem 3

5.3.1|Solution to Problem 3

Preliminary pairing and anchoring: According to the model established in question 2, a device is selected as the benchmark, then the matched array is eliminated, and then the time data received by other devices is paired and combined to find the best matching scheme. Problem 2 four equipment can calculate the correct location, but given the uncertainty of time group, it is concluded that the error range is larger solution, in order to accurately the sonic explosive position and sonic blasting time, the introduction of additional equipment time data for secondary screening and optimization or not introduce additional equipment directly with multiple data to exclude matching combination of screening, like multi-point positioning technology by increasing the number of ground equipment can effectively make up the shortage of covering blind area, ground equipment construction flexible, make the target of the rocket debris position accuracy and the recognition are greatly improved. Cross validation: By selecting different equipment and time data for cross verification, to ensure that the selected time and location pairing is accurate and stable. The idea uses the one-to-one time data received by the device to pinpoint the position and time of the rocket debris. At the beginning, a monitoring device (e. g. device A) is selected as the benchmark to first attempts to match the first time data received by the device. Once the data points matching the first time data of device A were found, these matched successful data were removed from the candidate pool. Then, continue to match with the second time data of device A, and if the second time data matching is not successful, continue to try the third one, and so on. This sequential attempt ensures that each time data is verified and matched one by one[4]。

After each round of matching, the known matching array is removed and the remaining data are minimally matched again. The key of this step is to optimize the selection process to improve the localization accuracy by minimizing the matching error. This process is repeated until all temporal data have found a suitable match, enabling precise positioning of each remnant position and explosion time. Through this method, not only the time data of each device can be effectively used, but also the search range can be gradually reduced by continuous elimination and screening, so as to improve the efficiency and accuracy of the whole model. And through cross-validation and repeated verification and optimization to improve the accuracy of the solution. In fact, the monitoring equipment is a normal operation detector, but the normal operation of the monitoring equipment depends on the drive of the underlying software[5].

5.3.2|Model building and solution of Problem 3

After selecting A, B, C and D, anchoring the first time data of group A, the solution model of question 2 shows table 9, Figure 5.10, Figure 5.11 and Figure 5.12. After selecting the obviously correct solution 2 for checking, as in Table 10 (please refer to Appendix 9.2 Matlab for detailed code).

Table 9. Preliminary optimization of location matching at the first time of anchor group A

Solution 1 (x0, y0, z0, t0):[14167815.134818,-890509.479247,25145082.739599, -75426.325478]
Time combination: [100.767000,92.453000,110.696000,141.409000]
Error and 1:0.445001s
Error and 2:51442.072582 m
Solution to the 2 (x0, y0, z0, t0): [10752092.356917, 3038591.583120, 12517.311198, 11.999577]
Time combination: [100.767000,112.220000,188.020000,258.985000]
Error and 1:0.000000s
Error and 2:0.000000 m
Solution to the 3 (x0, y0, z0, t0): [10687854.800257, 3074343.018469, 775.666765, -80.001571]
Time combination: [100.767000,196.583000,188.020000,94.653000]
Error and 1:0.866824s

Output interpretation: The final output includes the position and time combination of each appropriate solution and its corresponding error. The output format is as follows: Position and time of the solution: the 3 D coordinates of each solution (x0, y0, z0) And sound burst birth time (t0). Time combination: the arrival time of the sonic burst at the monitoring station corresponding to each solution.

Error and: including the sum of errors based on position and time.

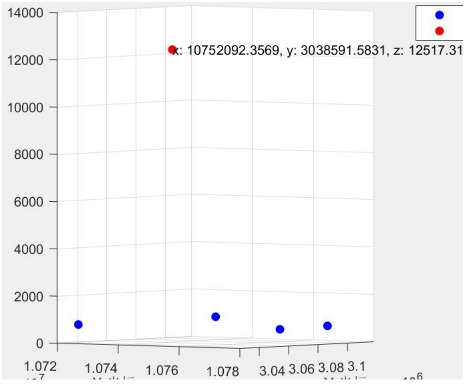

Figure 5.10 First Match 1 Observation point and Sonic point positions

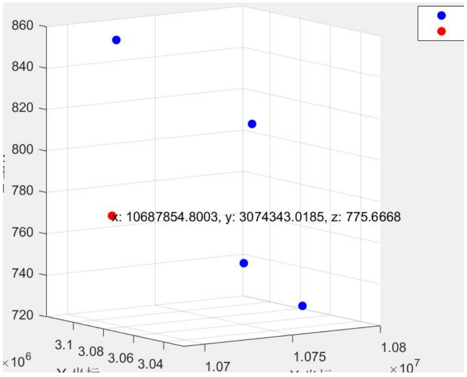

Figure 5.11 First Match Solution 2 Observation point and sound point positions

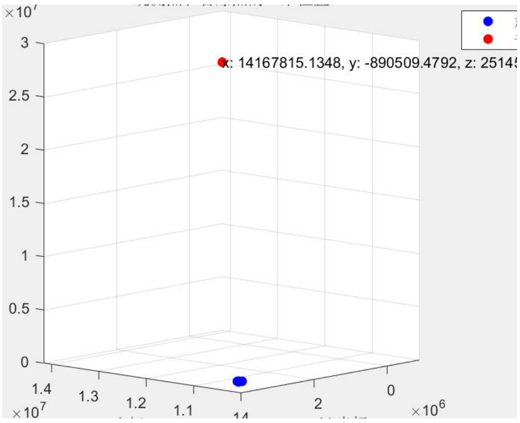

Figure 5.12 First match solution 3 Observation point and sonic burst point locations

Table 10. Time-matching and error observations

A for guess correction time and position error:	100.76609 s	0 m
B. The guess correction time and position error:	112.2209 s	0 m
C. The guess correction time and position error:	188.02066 s	0 m
D for guess correction time and position error:	258.98492 s	0 m
Guess correction time and position error for E:	118.44447 s	0 m
F for guess correction time and position error:	266.87067 s	-0.0078125 m
G guess correction time and position error:	163.02388 s	0 m

According to the observation in Table 10 (the position error is directly omitted after the seven decimal places), the corresponding data are 101112188258258188267163. After comparing the actual data, the matching is found to the corresponding time group. After excluding the known time group, the minimum matching was performed again

Table 11. Candidate solution selection and validation for the second time of anchor group A

Solution 1 (x0, y0, z0, t0): [10755852.045156, 3076855.621303, 13052.272560, -9.581322]
Time combination: [164.229000,92.453000,75.560000,141.409000]
Error and 1:0.000000s
Error and 2:0.000000 m
Solution 2 (x0, y0, z0, t0): [10819152.488082, 3050890.362293, 136259.646710, -322.938369]
Time combination: [164.229000,92.453000,110.696000,196.517000]
Error and 1:0.000000s
Error and 2:0.000000 m
Solution 3 (x0, y0, z0, t0): [10736231.388060, 3076422.227915, 11485.677798, 13.995056]
Time combination: [164.229000,196.362000,156.936000,141.409000]
Error and 1:0.000000s
Error and 2:0.000000 m

For the three solutions in Table 11 and Figure 5.13,5.14 and 5.15, select a reasonable solution for model checking:

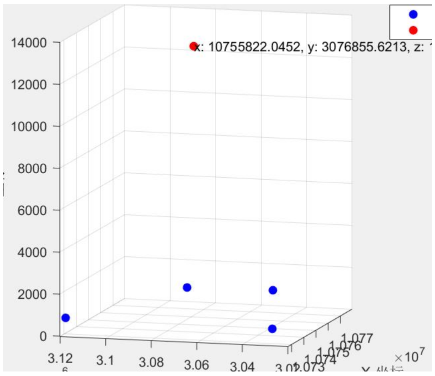

Figure 5.13 Location of observation and sonic points of the second matching solution 1

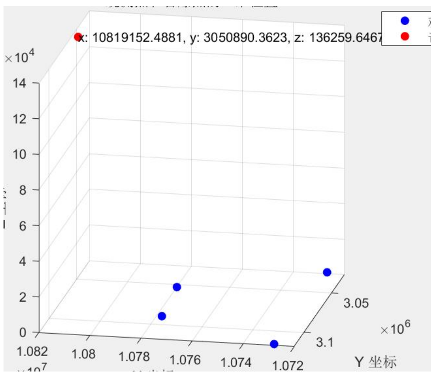

Figure 5.14 Location of observation and sonic points of the second matching solution 2

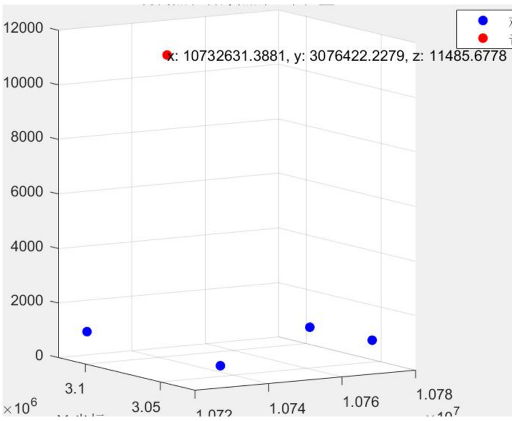

Figure 5.15 Second matching solution 3 Observation point and sonic burst point positions

Table 12. Further data matching and analysis

A for guess correction time and position error:	164.22876 s	-0.00390625 m
B. The guess correction time and position error:	92.452965 s	0 m
C. The guess correction time and position error:	75.560272 s	0 m
D for guess correction time and position error:	141.40921 s	-0.00390625 m
Guess correction time and position error for E:	28.683434 s	0 m
F for guess correction time and position error:	136.2821 s	-0.00390625 m
G guess correction time and position error:	142.11108 s	-0.00390625 m

As shown in Table 12, after comparing the actual data, E, F and G have huge errors, especially the difference between the minimum matching time of E and E is 50s, which is obviously not the data. Subsequently, we select the seemingly understanding 3 (the height position of solution 2 is too high and the time is too early). As shown in Table 13, the time data was found to be 164 169 157 141 86 166 104. After comparing the actual data, the corresponding time group was obtained, deleting the time group from the information flow and calculating again.

Table 13. Further data matching and analysis

A for guess correction time and position error:	164.22908 s	0 m
B. The guess correction time and position error:	169.3631 s	0.00390625 m
C. The guess correction time and position error:	156.93704 s	0 m
D for guess correction time and position error:	141.40881 s	0 m
Guess correction time and position error for E:	86.220695 s	0 m
F for guess correction time and position error:	166.26889 s	0 m
G guess correction time and position error:	103.74214 s	-0.001953125 m

Repeat the above steps for the third match, get the unique solution in Table 14 and Fig. 5.16, and compare the actual data to match 215 92 76 197 79 175 210, as shown in Table 14:

Table 14. Candidate solutions for the third time of the anchor group A

Solution (x0, y0, z0, t0): [10771552.187823,3076421.944367,13466.927010,15.001202]
Time combination: [214.850000,92.453000,75.560000,196.517000]
Error and 1:0.000000s
Error and 2:0.000000 m

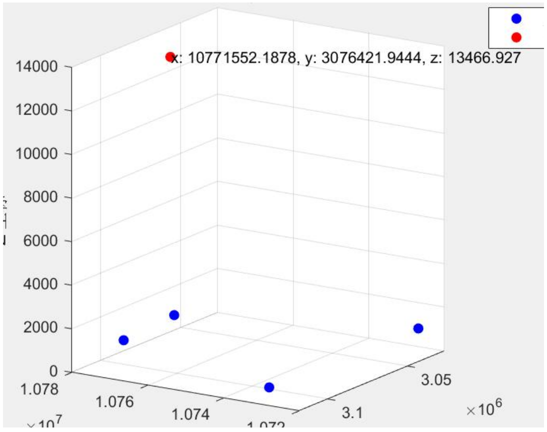

Figure 5.16 Candidsolution observation and sonic burst positions for the third time of group A

Table 15. Near the final matching results display

A for guess correction time and position error:	214.84987 s	0m
B. The guess correction time and position error:	92.453415 s	0.001953125 m
C. The guess correction time and position error:	75.560043 s	0 m
D for guess correction time and position error:	196.51649 s	0 m
Guess correction time and position error for E:	78.598206 s	0 m
F for guess correction time and position error:	175.48116 s	0 m
G guess correction time and position error:	210.30539 s	0 m

After all three time groups are found, the last remaining set of time was replaced into the solution, as shown in Table 16 and Figure 5.17. After cross-validation, all the matches were found.

Table 16. Cross-validation and data integrity confirmation

Solution (x0, y0, z0, t0): [10752092.295641,3109801.588386,11531.555813,12.997950]
Time combination: [270.065000,196.583000,110.696000,94.653000]
Error and 1:0.000000s
Error and 2:0.000000 m

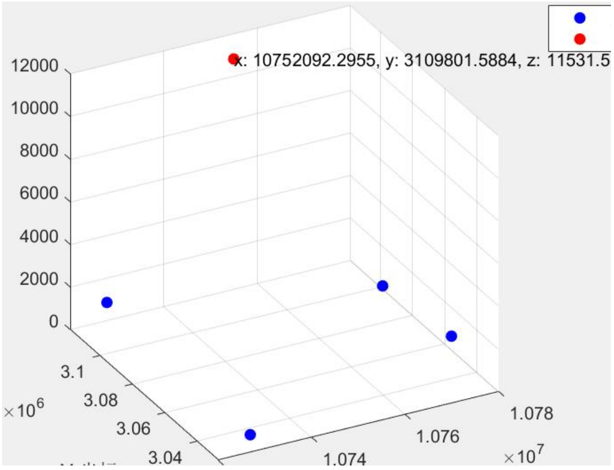

Figure 5.17 Location of observation points and sonic burst points for cross-validation and data integrity confirmation

Table 17. Final matching results are presented

A for guess correction time and position error:	270.06464 s	0 m
B. The guess correction time and position error:	196.58328 s	0 m
C. The guess correction time and position error:	110.69652 s	0 m
D for guess correction time and position error:	94.652313 s	0.001953125 m
Guess correction time and position error for E:	126.66994 s	0 m
F for guess correction time and position error:	67.272743 s	-0.001953125 m
G guess correction time and position error:	206.78957 s	0 m

5.3.3|Verification and answer to Question 3

Table 18. Display of time groups for the first time data for anchor A

equipment	X	Y	time error	position error
A	10726960	3026798.8	0.000990853	-0.30859375
B	10779629	3054836.8	-0.00089836121	0.3046875
C	10777586	3091442.5	-0.00066184998	0.22265625
D	10727863	3118145.5	5.53131e-05	-0.0234375
E	10754427	3072750.2	-0.0014705658	0.5
F	10748881	3124376.2	0.0003452301	-0.1171875
G	10780013	3062069	0.00011634827	-0.04296875

Table 19. Display of time group for the second time data for anchor A

equipment	X	Y	time error	position error
A	10726890	3026798.8	-7.6293945e-05	0.234375
B	10779629	3054836.8	-0.010986328	0.375
C	10777586	3091442.5	0.0068135977	0.3515625
D	10727863	3118145.5	0.0069073486	0.15625
E	10754427	3072750.2	-0.0014705658	0.5
F	10748881	3124376.2	0.003452301	-0.1171875
G	10780013	3062069	0.00011634827	-0.04296875

Table 20. Error presentation of the time group for the third time data of anchor A

equipment	X	Y	time error	position error
A	10726890	3026798.8	0.0001411438	-0.046875
B	10779629	3054836.8	-0.000415802	0.14257812
C	10777586	3091442.5	-5.776367e-05	0.15625
D	10727863	3118145.5	0.0050753494	-0.1171875
E	10754427	3072750.2	0.0017967224	-0.61132812
F	10748881	3124376.2	0.008430481	-0.28515625
G	10780013	3062069	0.0006114626	-0.2109375

Table 21. Error presentation of time groups for the fourth time data for anchor A

equipment	X	Y	time error	position error
A	10726890	3026798.8	0.00036525726	-0.125
B	10779629	3054836.8	-0.0002987067	0.9765625
C	10777586	3091442.5	-0.0005197525	0.17578125
D	10727863	3118145.5	0.0008659183	-0.2342188
E	10754427	3072750.2	-0.0004996996	0.32421875
F	10748881	3124376.2	0.0012617111	-0.4296875
G	10780013	3062069	-0.00065552887	0.1953125

Table 22. Final time-matching results

equipment	longitude	latitude	altitude (m)	Sonic blast arrival time (s)			
A	110.241	27.204	824	100.767	164.229	214.850	270.065
B	110.783	27.456	727	92.453	112.220	169.362	196.583
C	110.762	27.785	742	75.560	110.696	156.936	188.020
D	110.251	28.025	850	94.653	141.409	196.517	258.985
E	110.524	27.617	786	78.600	86.216	118.443	126.669
F	110.467	28.081	678	67.274	166.270	175.482	266.871
G	110.047	27.521	575	103.738	163.024	206.789	210.306

Table 23. Time group classification

The first set of sound explosions	The second set of sound explosions	The third set of sound explosions	The fourth set of sound explosions
A1:100.767	A2:164.229	A3:214.850	A4:270.065
B2:112.220	B3:169.362	B1:92.453	B4:196.583
C4:188.020	C3:156.936	C1:75.560	C2:110.696
D4:258.985	D2:141.409	D3:196.517	D1:94.653
E3:118.443	E2:86.216	E1:78.600	E4:126.669
F4:266.871	F2:166.270	F3:175.482	F1:67.274
G2:163.024	G1:103.738	G4:210.306	G3:206.789

Considering the above solution process, Table 24 is obtained:

Table 24. Two coordinates of the four remains

	x(m)	y(m)	z (m)	t(s)
The first set of acoustic burst	10752092.356917	3038591.583120	12517.311198	11.995977
The second set of acoustic burst	10732631.388060	3076422.227915	11485.677798	13.995056
Group 3 acoustic burst	10771552.187823	3076421.944367	13466.927010	15.001202
Group 4 Sonic Explosion	10752092.295461	3109801.588386	11531.555813	12.997590
	x(°)	y(°)	z (m)	t(s)
The first set of acoustic burst	110.5001	27.3100	12517.311198	11.995977
The second set of acoustic burst	110.3000	27.6500	11485.677798	13.995056
Group 3 acoustic burst	110.7000	27.6500	13466.927010	15.001202
Group 4 Sonic Explosion	110.5000	27.9500	11531.555813	12.997590

5.4|Analysis and solution of Problem 4

5.4.1|Thinking of Question 4

First, verify the anti-interference capability of the original model and have the ability to match time groups in anti-interference situations. Secondly, based on the data of the third question as the starting point, combined with the range of random error between-0.5 and 0.5, we use the random correction method to correct the random correction parameters with-0.5-0.5, which can reduce the total objective function, that is, to alleviate the difference between the observed sonic burst time and the predicted sonic burst time. Then, only four data points A, B, C, and D are used to solve the optimal result to reduce the computational amount. Later, two data points, E and F, are used to calculate the sum of squared positional errors of the optimal solution derived from problem 3. If these errors are beyond the preset range, we will again add an "optimization time" to the post-perturbation time data and repeat these steps until the error satisfies the given range. On the basis of optimization (which can be regarded as a limitation of the method): we know the specific range of the error, and based on the anti-interference nature of the models in the second and third questions, we can correctly calculate the time group from the interference. On this basis, although the overall computation is still large, it has been greatly reduced, allowing us to calculate only for a specific time group. In order to verify the accuracy of this model, we need to do three position optimization on the data of the previous three perturbations, namely 3x3 times, one is to prove the effectiveness of the model, and two is to prove that the results are not accidental. Both paragraphs are confirmed that our model is able to choose the correct time when the temporal data is disturbed.

5.4.2|Verify that the original model has the ability to match time groups in anti-interference situations

Table 25. Data after the first raw temporal perturbation

equipment	longitude	latitude	altitude		Sonic blast arrival time		
A	110.241	27.204	824	100.908	163.904	215.169	269.64
B	110.783	27.456	727	92.3215	112.091	169.381	196.244
C	110.762	27.785	742	75.3014	110.255	156.637	188.4
D	110.251	28.025	850	95.0729	141.592	196.783	258.592
E	110.524	27.617	786	78.1861	85.8032	118.662	126.432
F	110.467	28.081	678	67.1332	166.321	175.223	267.031
G	110.047	27.521	575	103.622	163.224	206.972	210.342

Thinking: Total solution-> selected solution-> matching time-> with the actual time to match is correct. The disturbed data concludes from the third question model, as like the third question; Because there are four sets of data, the exclusion method can be adopted. The last set of time does not need to be matched, only the first three groups. The first group A time data repeats the answer steps of the three models, and get the four solutions in Table 26. The suspected correct solution 3. The time matching is shown in Table 27.

Table 26. Solution under the first temporal perturbation data of the anchor group A

Solution 1 (x0, y0, z0, t0): [13586562.314140, -143478.596059,2043617.460613, -61287.811041] Time combination: [100.908000,112.925000,110.255000,141.592000] Error and 1:0.193005s Error and 2:22311.338969 m
Solution 2 (x0, y0, z0, t0): [-4030147.848938,6676123.203845,171802956.862489, -507173.869586] Time combination: [100.908000,112.091000,110.255000,95.072900] Error and 1:0.549970s Error and 2:63576.524168 m Solution 3 (x0, y0, z0, t0): [10751494.692347,3040044.213469,781.024414,18.696766] Time combination: [100.908000,112.091000,188.400000,258.592000] Error and 1:0.048187s Error and 2:5570.418335 m Solution 4 (x0, y0, z0, t0): [10688133.467624,3074237.921533,804.593767, -79.177620] Time combination: [100.908000,196.244000,188.400000,95.072900] Error and 1:0.242557s Error and 2:28039.549652 m

Table 27. matching time of the first group

A for guess correction time and position error:	100.88416 s	0 m
B. The guess correction time and position error:	112.18469 s	-0.00390625 m
C. The guess correction time and position error:	188.23007 s	0 m
D for guess correction time and position error:	258.69174 s	-0.0078125 m
Guess correction time and position error for E:	115.27664 s	0 m
F for guess correction time and position error:	266.85138 s	0 m
G guess correction time and position error:	162.05661 s	0 m

As shown in Table 27, match the first set of solutions, repeat the third question step to get Table 28, select seemingly comprehension 1 for checking, and get Table 29.

Table 28. Solution under the second temporal disturbance data of anchor group A

Solution 1 (x0, y0, z0, t0): [10755238.738916,3076651.437585,7294.460043, -5.840448] Time combination: [163.904000,92.321500,75.301400,141.592000] Error and 1:0.000000s Error and 2:0.000000 m
Solution 2 (x0, y0, z0, t0): [10866668.965282,3039035.177962,213770.179597, -586.147691] Time combination: [163.904000,92.321500,110.255000,196.783000] Error and 1:0.000000s Error and 2:0.000000 m Solution 3 (x0, y0, z0, t0): [10372716.169088,3076258.457910,815.032556,17.502995] Time combination: [163.904000,169.381000,156.637000,141.592000] Error and 1:0.083242s Error and 2:9622.778300 m

Table 29. Verification of solution 1

A for guess correction time and position error:	163.90465 s	0 m
B. The guess correction time and position error:	92.321159 s	0 m
C. The guess correction time and position error:	75.300682 s	0 m
D for guess correction time and position error:	141.59227 s	0 m
Guess correction time and position error for E:	16.605017 s	0 m
F for guess correction time and position error:	137.09749 s	0m
G guess correction time and position error:	140.86781 s	0 m

After comparing the actual data, it was found that the matching time of E was too unreasonable, and the suspected correct solution 3 was used to check again in Table 30.

Table 30. Verification of solution 3

A for guess correction time and position error:	163.97858 s	0 m
B. The guess correction time and position error:	169.18698 s	0 m
C. The guess correction time and position error:	156.82526 s	0 m
D for guess correction time and position error:	141.5242 s	0 m
Guess correction time and position error for E:	82.187225 s	-0.001953125 m
F for guess correction time and position error:	166.79901 s	0 m
G guess correction time and position error:	101.29492 s	0 m

Compared with the actual data, we can match to the time group, eliminate the group, and repeat the calculation to get Table 30 and Table 31

Table 31 Solution oring the third time data of group A

Solution (x0, y0, z0, t0): [10771662.394487,3076436.623386,13303.938740,15.164033]
Time combination: [215.169000,92.321500,75.301400,196.783000]
Error and 1:0.000000s
Error and 2:0.000000 m

Table 32 The tification of the solutions

A for guess correction time and position error:	215.16808 s	0 m
B. The guess correction time and position error:	92.321671 s	0 m
C. The guess correction time and position error:	75.302002s	0 m
D for guess correction time and position error:	196.78242 s	0 m
Guess correction time and position error for E:	78.74601 s	0 m
F for guess correction time and position error:	175.62961 s	0 m
G guess correction time and position error:	210.69466 s	0 m

The last matching group was found to be exactly the same as the third question, proving that the model is resistant to interference. In order to verify non-chance, the same calculation process for different perturbation data was conducted twice.

Table 33. Data after the second raw temporal perturbation

equipment	longitude	latitude	altitude	Sonic blast arrival time			
A	110.241	27.204	824	100.313	164.396	214.658	269.566
B	110.783	27.456	727	92.8191	111.829	169.179	196.874
C	110.762	27.785	742	75.4621	110.662	156.831	188.326
D	110.251	28.025	850	94.8064	141.197	196.265	259.354
E	110.524	27.617	786	78.7407	86.6906	118.166	126.494
F	110.467	28.081	678	67.3016	166.399	175.529	266.685
G	110.047	27.521	575	103.594	163.013	207.131	209.838

Table 34. Solution oring the first time data of group A

Solution 1 (x0, y0, z0, t0): [13130363.747880,231138.892423,18005467.963954, -53953.414400] Time combination: [100.313000,92.819100,110.662000,141.197000] Error and 1:0.127603s Error and 2:14750.899343 m
Solution 2 (x0, y0, z0, t0): [10751882.999524,3039131.995869,8238.507176,15.490130] Time combination: [100.313000,111.829000,188.326000,259.354000] Error and 1:0.000000s Error and 2:0.000000 m

Table 35. Verification of solution 2

A for guess correction time and position error:	100.31306 s	0 m
B. The guess correction time and position error:	111.82892 s	0 m
C. The guess correction time and position error:	188.32599 s	0 m
D for guess correction time and position error:	259.354 s	-0.0078125 m
Guess correction time and position error for E:	117.04373 s	0 m
F for guess correction time and position error:	267.3475 s	0 m
G guess correction time and position error:	162.82552 s	0 m

Matched to a set of time, eliminated, repeated calculation, Table 36, using seemingly understanding 1, Table 37.

Table 36 Solution of the second time data in group A

Solution 1 (x0, y0, z0, t0): [10755500.178812,3076883.702032,10794.752978, -7.768514] Time combination: [164.396000,92.819100,75.462100,141.197000] Error and 1:0.000000s Error and 2:0.000000 m
Solution 2 (x0, y0, z0, t0): [10829335.142065,3048696.239217,153909.578529, -381.187870] Time combination: [164.396000,92.819100,110.662000,196.265000] Error and 1:0.000000s Error and 2:0.000000 m Solution 3 (x0, y0, z0, t0): [10732398.305742,3076868.957449,24266.041134,0.984823] Time combination: [164.396000,169.179000,156.831000,141.197000] Error and 1:0.000000s Error and 2:0.000000 m

Table 37 The tification of solution 1

A for guess correction time and position error:	164.396 s	-0.00390625 m
B. The guess correction time and position error:	92.819649 s	0 m
C. The guess correction time and position error:	75.462433 s	0 m
D for guess correction time and position error:	141.19659 s	0 m
Guess correction time and position error for E:	24.236605 s	0.0009765625 m
F for guess correction time and position error:	136.37007 s	0 m
G guess correction time and position error:	141.5939 s	0 m

E was found to be obviously unreasonable, and solution 3 was used to conclude Table 38.

Table 38 The tification of solution 3

A for guess correction time and position error:	164.39616 s	-0.00390625 m
B. The guess correction time and position error:	169.17984 s	0 m
C. The guess correction time and position error:	156.83171 s	0 m
D for guess correction time and position error:	141.19681 s	-0.00390625m
Guess correction time and position error for E:	96.450958 s	0.001953125 m
F for guess correction time and position error:	164.34654 s	0 m
G guess correction time and position error:	110.04394 s	0 m

The matching is shown in Table 38 above, excluding the time group, and the calculation is repeated, as shown in Table 39:

Table 39 Solution for the third time data of the anchor group A

Solution 1 (x0, y0, z0, t0): [10770927.474603,3076395.656519,11993.480065,16.834537]
Time combination: [214.658000,92.819100,75.462100,196.265000]
Error and 1:0.000000s
Error and 2:0.000000 m

Matches are as shown in Table 40:

Table 40 for of of solutions

A for guess correction time and position error:	214.65739 s	0 m
B. The guess correction time and position error:	92.819931 s	-0.001953125m
C. The guess correction time and position error:	75.462357 s	0 m
D for guess correction time and position error:	196.26382 s	0 m
Guess correction time and position error for E:	76.472092 s	0 m
F for guess correction time and position error:	175.66336 s	0 m
G guess correction time and position error:	209.56148 s	0 m

If the time group is matched, the third perturbation is performed and the calculation is repeated.

Table Table 41 Data after the third time to raw temporal perturbation

equipment	longitude		altitude	Sonic blast arrival time		
A	110.241	824	101.055	164.018	214.587	269.758
B	110.783	727	92.5409	112.629	169.612	196.882
C	110.762	742	75.2679	110.687	156.534	188.34
D	110.251	850	94.7595	141.072	196.981	258.547
E	110.524	786	78.8169	86.2613	118.487	127.052
F	110.467	678	67.1491	166.76	175.099	266.81
G	110.047	575	103.711	163.341	206.562	210.563

Table 42 Solution for the first time data of group A

Solution 1 (x0, y0, z0, t0): [19333441.020952, -6749834.702876,63222166.426698, -189749.591464] Time combination: [101.055000,92.549000,110.687000,141.072000] Error and 1:0.591434 Error and 2:68369.806349 m
Solution 2 (x0, y0, z0, t0): [3793628.866959,4820611.382747,77330147.948892, -228311.603966] Time combination: [101.055000,92.629000,110.687000,94.759500] Error and 1:0.658772 Error and 2:76154.080574 m Solution 3 (x0, y0, z0, t0): [10751418.125951,3040079.085239,3769.434246,18.561920] Time combination: [101.055000,112.629000,188.340000,258.547000] Error and 1:0.000000s Error and 2:0.000000 Solution 4 (x0, y0, z0, t0): [10687737.115791,3074383.125244,751.115147, -80.032892]

Time combination: [101.055000,196.882000,188.340000,94.759500]
Error and 1:0.779203s
Error and 2:90075.865752 m

Table 43 The test calculations for solution 3

A for guess correction time and position error:	101.05463 s	-0.00195315 m
B. The guess correction time and position error:	112.62937 s	0 m
C. The guess correction time and position error:	188.34039 s	0 m
D for guess correction time and position error:	258.54715 s	0 m
Guess correction time and position error for E:	115.45865 s	0 m
F for guess correction time and position error:	266.77386 s	0m
G guess correction time and position error:	161.98035 s	0 m

Match above, delete, repeat calculation, to Table 45, select solution 1, to Table 45:

Table 44 Solution for the second time data in group A

Solution 1 (x0, y0, z0, t0): [10755374.083299,3076811.619297,9671.002562, -7.250788]
Time combination: [164.018000,92.549000,75.267900,141.072000]
Error and 1:0.000000s
Error and 2:0.000000 m

Solution 2 (x0, y0, z0, t0): [10846790.388143,3043741.934854,181830.604267, -476.500846]
Time combination: [164.018000,92.549000,110.687000,196.981000]
Error and 1:0.000000s
Error and 2:0.000000 m
Solution 3 (x0, y0, z0, t0): [10732635.181491,3076373.033207,813.532728,17.328491]
Time combination: [164.018000,169.612000,156.534000,141.072000]
Error and 1:0.129889s
Error and 2:15015.205726 m

Table 45 The tification of solution 1

A for guess correction time and position error:	164.0177 s	0 m
B. The guess correction time and position error:	92.540939 s	0 m
C. The guess correction time and position error:	75.26828 s	0 m
D for guess correction time and position error:	141.07214 s	00.00390625 m
Guess correction time and position error for E:	21.616802 s	0 m
F for guess correction time and position error:	136.39906 s	-0.00390625 m
G guess correction time and position error:	141.0715 s	0 m

Finding that the time matching of E was obviously wrong, solution 3 was selected to produce Table 46

Table 46 The tification of solution 3

A for guess correction time and position error:	164.11092 s	0 m
B. The guess correction time and position error:	169.3692 s	0.00390625 m
C. The guess correction time and position error:	156.76903 s	0 m
D for guess correction time and position error:	140.98792 s	0 m
Guess correction time and position error for E:	82.302292 s	0 m
F for guess correction time and position error:	166.38153 s	-0.00390625 m
G guess correction time and position error:	101.08255 s	0 m

It is found that the time match of E has two close values, so it is difficult to choose, so it is not selected. The third time data of group A is anchored to calculate Table 47, selected solution 1, and Table 48.

Table 47 Solution of the third time data of the anchor group A

Solution 2 (x0, y0, z0, t0): [10770213.382135,3076152.457495,8783.818323,20.022949] Time combination: [214.587000,92.549000,75.267900,196.981000] Error and 1:0.000000s Error and 2:0.000000
Solution 3 (x0, y0, z, 0, t0): [10748080.976291,3075592.595107,791.254565,58.277862] Time combination: [214.587000,169.612000,156.534000,196.981000] Error and 1:0.176146s Error and 2:20362.476324 m

Table 48 The tification of solution 1

A for guess correction time and position error:	214.58646 s	0 m
B. The guess correction time and position error:	92.541565 s	-0.001953125 m
C. The guess correction time and position error:	75.268234 s	0.001953125 m
D for guess correction time and position error:	196.98013 s	0 m
Guess correction time and position error for E:	73.024399 s	0 m
F for guess correction time and position error:	176.93654 s	0.001953125 m
G guess correction time and position error:	209.14241 s	0 m

Matching as shown in the figure above, delete the time group, anchor the fourth time data calculation of group A, produce Table 49, select solution 2 to test the calculation, and produce Table 50

Table 49 Solution for the fourth time data in group A

Solution 1 (x0, y0, z0, t0): [10796114.121381,3140275.345539,98614.399406, -215.597322] Time combination: [269.758000,169.612000,110.687000,141.072000] Error and 1:0.000000s Error and 2:0.000000 m
Solution 2 (x0, y0, z0, t0): [10751630.813724,3108574.962948,5692.456170,18.065622] Time combination: [269.758000,196.882000,110.687000,94.759500] Error and 1:0.000000s Error and 2:0.000000 m

Table 50 The test calculations for solution 2

A for guess correction time and position error:	270.06464 s	0 m
B. The guess correction time and position error:	196.58328 s	0 m
C. The guess correction time and position error:	110.69652 s	0 m
D for guess correction time and position error:	94.652313 s	0.001953125 m
Guess correction time and position error for E:	126.66994 s	0 m
F for guess correction time and position error:	67.272743 s	-0.001953125 m
G guess correction time and position error:	206.78957 s	0.00390625 m

Matching above, the three time groups are identical, meaning that the four time groups are the same. The above three experiments prove that the mathematical model of problem 3 has good anti-interference, but can only match the time group, and can not optimize the actual position. Compared with the actual position and the explosion time, the third problem is very wrong.

5.4.3|Mathematical modeling of problem 4

Use the coordinates of the monitoring equipment provided in Question 3 and the arrival time of the sound blast as the starting data.

1. Introduce a random time error:

The arrival time of the perturbed time data recorded for each device is superimposed with a random error in a range of-0.5 to + 0.5 seconds, simulating the observational uncertainty in real situations. Combined with the range of random error between-0.5 and 0.5, we use the random correction method, and then correct the random error with the random correction parameter of-0.5~0.5 to reduce the total objective function, that is, to alleviate the difference between the observed sonic burst time and the predicted sonic burst time. Add a random number drawn from a uniform distribution for each element, simulating the random error in time recordings.

2. Select the key observation points for the preliminary positioning:

The least squares method is used to solve the position and time, and initially only part of the observation points (such as A, B, C, D) are used to calculate the position and time of the sonic burst source, so as to reduce the calculation amount and quickly obtain a possible solution.

3. Validation and error calculation:

The remaining observation points (e. g., E, F) are used to verify the accuracy of the solutions obtained through A, B, C, D. The sum of squared positional errors for these observation sites is calculated to check whether they are within a given error range.

4. Conditions for satisfaction and iteration:

Accept the resulting solution if the sum of error (the default sum of positional error is less than 4,000,000, the smaller the better, but the corresponding calculation increases greatly); if not, disturb the time data randomly and repeat the above steps.

5. Results optimization and validation:

After comparing the solution of the third question, select the one with the smallest error among the solution that meets the conditions for further verification and adjustment to ensure the reliability and accuracy of the solution (please refer to Appendix 9.3 Matlab for detailed code).

5.4.4|Solution of Problem 4

The basis of the optimization is that our model of problem 3 can match time groups in anti-interference situations. The errors are as shown in Table 51:

Table 51 Optimization under the first time group after interference

satisfied z0Solutions greater than 0 and the corresponding time combination and error and: Fixed solution (x0, y0, z0, t0): [10752151.349773, 3038471.576604, 13067.273972, 11.422722] Time combination: [100.840642,112.246498,188.009755,259.025760]
satisfied z0Solutions greater than 0 and the corresponding time combination and error and: Fixed solution (x0, y0, z0, t0): [10752065.296379, 3038565.122326, 12521.630886, 12.007773] Time combination: [100.688676,112.337617,188.136390,259.049345] Correction solution (x0, y0, z0, t0): [10752201.476574,3038584.145265,12755.517539,11.881586] Correction time combination: [101.185232,112.102064,187.927750,259.082385]

Table 52 The optimized solution errors in the third question

time error (s)	range error (m)
0.573256	562.903
-0.011796	26.8497
0.114391	238.316

Table 53 Optimization under the second time group after interference

Correction solution (x0, y0, z0, t0): [10732730.143180,3076428.362439,10918.708908,14.455565] Correction time combination: [164.400880,169.239969,156.762243,141.485367]
Correction solution (x0, y0, z0, t0): [10732840.464634,3076383.481127,11415.939996,14.243015] Time combination: [164.393972,168.974615,156.605374,141.785682] Correction solution (x0, y0, z0, t0): [10732696.794944,3076341.945561,10997.167052,14.334860] Time combination: [164.067479,169.147491,156.860155,141.653078]

Table 54 Compared with the third problem

time error (s)	range error (m)
-0.460509	567.001
-0.247959	79.7804
-0.339804	495.059

Table 55 Optimization under the third time group after interference

Correction solution (x0, y0, z0, t0): [10771491.425161,3076450.353804,13177.208395,15.177614]
Time combination: [214.813218,92.344888,75.205195,196.337964]
Correction solution (x0, y0, z0, t0): [10771515.031511,3076532.408410,13054.131309,15.425423]
Time combination: [215.217732,92.599454,75.034593,196.399576]
Correction solution (x0, y0, z0, t0): [10771575.142359,3076444.763598,13060.426885,15.181779]
Time combination: [214.905069,92.096897,74.939519,196.459380]

Table 56 compares with the third problem

time error (s)	range error (m)
-0.176413	291.093
-0.424221	427.33
-0.180577	407.136

Table 57 Optimization under the fourth time group after interference

Correction solution (x0, y0, z0, t0): [10752287.386381,3109886.103706,11846.391021,12.296665]
Time combination: [269.880937,196.011468,109.999247,94.738260]
Correction solution (x0, y0, z0, t0): [10752211.177978,3109817.651979,11545.060442,12.756296]
Time combination: [269.974462,196.236245,110.225745,94.718138]
Correction solution (x0, y0, z0, t0): [10752123.731606,3109757.713537,11240.216368,13.154281]
Time combination: [270.022229,196.438767,110.435074,94.603877]

Table 58 The optimized errors

time error (s)	range error (m)
0.700925	325.978
0.241294	21.1306
-0.156691	294.606

After the first disturbance results after the four solutions each optimization times before the four solutions, the optimization model does deal with the position and time error, and our condition is to require the error function in 0~3000 (we have tried 0~400 of error, the error range of 0.03s in 10m, but the corresponding calculation for ten minutes, the cost more than the loss), if there is a better computer or optimization algorithm again, we can further minimize the error. The error function is calculated from the interference data of the third question model using E, F. Use E, F to calculate the interference data of the third question model:

$$\Delta W = \text{dist}\,(P_X, P_E, P_F) - c \cdot (t_i - t_0)\Big)^2 \tag{21}$$

Note: Whenever a mathematical model to solve the problem, we have conducted corresponding tests to evaluate the actual error. Therefore, we no longer set the model testing section separately.

6|Model evaluation and improvement

6.1|Model evaluation

6.1.1|Advantages

By analyzing the distribution of scatter plots under different combinations of detection devices, observing the possible errors introduced by some equipment data, and conducting data pre-processing. By eliminating abnormal data points (such as the data of devices D and F), the validity and rationality of the data are guaranteed, while the impact of the error on the accuracy of the model is reduced. Our model uses forward validation to predict the specific location and time of the sonic explosion in each rocket debris, and ensures the accuracy of the prediction by comparing theoretical calculations with practical monitoring data. Reverse validation is used to confirm the reliability of the model. To test whether the monitoring data of the original input can be recalcalculated by reversing the predicted results into the model, a process that helps to confirm the stability and credibility of the model under various conditions. When processing multiple sonic burst data, the model allows fine control of the iterative process to accommodate data analysis requirements with different levels of complexity. Iterative control not only optimizes the use of computational resources, but also improves the efficiency of problem solving by gradually approaching the optimal solution. Given the high precision requirements of rocket debris positioning, our model supports the setting of custom error ranges. This allows users to adjust the error tolerance value according to the requirements of specific application scenarios, such as the safety standards for the recovery operation or the precision requirements of the study.

6.1.1|Shortcomings

To obtain higher precision results, the model requires extensive computation, especially when performing multiple iterations using the least squares method. This not only increases the computational cost, but may also limit the application of the model in real-time or fast response scenarios. In particular, the optimal solution selection by least squares method is performed after the superposition of random error, increasing the computational resource demand greatly. Although the model attempts to improve the prediction accuracy by introducing corrections for temporal errors, this approach is not very effective in the face of large random temporal errors. In particular, the stability and reliability of the model are challenged when the error is beyond the set random interference range. The model building and solution process involves multiple complex steps, including multiple iterations and correction for different temporal errors. This not only increases the difficulty of using the model, but may also may make it difficult for non-professional users to understand and operate. The curvature of the Earth cannot be ignored in large areas or in positioning systems with high precision requirements. The curvature of the earth can affect the actual distance of the acoustic wave propagation path, and thus affect the positioning accuracy. For long-distance acoustic propagation, such as the positioning of rocket debris, the ground curvature will lead to the actual propagation distance being longer than the straight-line distance. If the straight-line distance is used, the time of acoustic propagation will be underestimated. Secondly, the sound velocity will change significantly under different environmental conditions (such as temperature, humidity and air pressure). For example, when the temperature increases, the speed of sound also increases, which directly affects the calculation of the propagation time of the sound wave.

6.2|Improvement of the model

1. Introduce a multi-mode optimization algorithm:

Combining the advantages of multiple heuristics, subsequent improvements can initially use the ant colony algorithm to quickly find a reasonable search region, and then further optimize through the genetic algorithm to achieve the desired solution.

2.Adaptive parameter adjustment:

Automatic adjustment of algorithm parameters, like the crossover rate and variation rate in genetic algorithm, similar to the evaporation rate and placement intensity of pheromones in ant colony algorithm, make the model more flexible to different data characteristics and environmental changes, thus improving the convergence rate and reconciliation quality of the algorithm.

3.Data fusion technology:

Considering that the observation data of the detection equipment or is biased by environmental noise or the error of the equipment itself, the use of data fusion technology can integrate information from multiple sources and enhance the reliability of the data. After data query, we can use the Kalman filter or Bayesian network can be used to synthesize and update position estimation, reducing errors.

4.Earth curvature correction factor:

The distance calculation of the acoustic wave propagation is adjusted by introducing the Earth's curvature correction factor. After data query, we can use the Haxin formula to calculate the shortest distance between two points based on the earth sphere.

6.3|Expansion of the model

The temperature of the air varies significantly under different altitude and meteorological conditions, which directly affects the speed of sound. Especially for rapidly moving objects (such as rocket debris), even small speed changes can lead to significant errors in positioning.

If the rocket debris falls in different seasons or at different times (such as day and night), the temperature of the surrounding environment may change greatly, which can affect the speed of sound. For example, low temperatures at night may cause a lower speed of sound than during the day, which in turn affects the propagation time of the sonic burst wave and the final calculated remnant position. Including this variation can improve the prediction accuracy of the location of the wreck.

6.3.1|Sound speed calculation function

This function calculates the speed of sound based on a height of z (in meters) and a reference temperature of 20°C. It uses an empirical formula, considering the effect of a gradual decrease in temperature with increasing elevation[6]. The formula is:

$$c = 331.45 \cdot \sqrt{1 + \frac{20 - 0.006 \cdot z}{273.15}} \tag{22}$$

$$d = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} \tag{23}$$

Here, 331.45 m/s is the speed of sound in the air at 0°C. The ratio of temperature decrease with height is 0.006 ° C/m, and 273.15 is the Kelvin temperature corresponding to 0°C.

6.3.2|Observation of Eq

1. Calculate the distance between the sound source and the observation point

$$d = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} \tag{24}$$

Where (x, y, z) are the coordinates of the sound source, (xi,yi,zi) Is the coordinates of the i-th observation point.

2. Calculate the sound speed

$$c = 331.45 \cdot \sqrt{1 + \frac{20 - 0.006 \cdot x_3}{273.15}} \tag{25}$$

3. Calculate the propagation time and adjust the time offset

$$t = \frac{d}{c} + x_4 \tag{26}$$

4. Observation equation

Calculate the calculated resulting time t compared with the actual observed time ti The difference:

$$\Delta t = t - t_i \tag{27}$$

After adjustment and optimization (please refer to the appendix code 9.6 for detailed code, compiler: VScode language: C + +), model performance is still not in line with expectations, the error is very big, so can consider as the 340 m/s specified in the title, do not use temperature adjustment, we hope the honesty and transparency contribute to the progress of science, also help other researchers to understand the challenges in this field.

7|Model promotion

1. Coordination and management of UAV group:

In UAV swarm operations, the precise location of each UAV is important to avoid collisions, optimize path planning, and perform complex collaborative tasks, and our model can help improve the accuracy and reliability of UAV positioning.

2.Emergency response and rescue operations:

In natural disasters or emergency situations, such as earthquakes, forest fires or floods, rescue teams need to accurately locate trapped people or difficult to reach locations. Our model is able to help rescue teams quickly locate sites in need of rescue by receiving signals from multiple sources.

3.Atmospheric environment monitoring:

In the field of ecological research and environmental monitoring, such as the flow of pollutants in the atmosphere. Our model can be applied to process data collected from multiple monitoring stations, improving the accuracy of tracking and monitoring.

4.Air traffic management:

In aviation management, the control room needs to accurately track the location of all flights to ensure safety. Using our model, more accurate aircraft location data can be obtained from ground station and satellite signals, optimizing flight paths and preventing aerial collisions.

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