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Study on Noise Analysis and Frequency Estimation in Aircraft Laser Speed Measurement Technology

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Abstract

Aiming at the problem of accuracy degradation of laser velocimetry signals in high-speed flight scenarios due to complex noise interference, this study constructs a frequency estimation method based on the noise feature analysis of Butterworth filter and multi-domain spectrum fusion. Through the quantitative analysis of noise mean, variance and power spectral density, combined with the joint spectral analysis of Fourier Transform (FT) and Discrete Fourier Transform (DFT), the high-precision frequency estimation of noise-containing signals is achieved in MATLAB simulation environment, and the estimated frequency is 40999836.00Hz, with the relative error less than 0.01%. The proposed algorithm improves the signal-to-noise ratio by more than 35% while maintaining the signal integrity, which provides a reliable theoretical and methodological support for the noise suppression and parameter measurement of the laser speed measurement system under the high dynamic flight environment.

Keywords: Laser Doppler Velocimetry; Noise Suppression; Spectral Analysis; Butterworth Filter; Velocimetry Accuracy

1 | Introduction

The accuracy of airspeed measurement is a core indicator for flight safety and aerodynamic performance optimisation, especially in high-speed manoeuvre scenarios (e.g., large overload flights of fighter jets), where a 0.1% error in velocity measurement may lead to a deviation of more than 5° in the calculation of angle of attack, which can increase the risk of stalling significantly [1]. Laser Doppler velocimetry has become the core means of airspeed measurement for high-speed vehicles by virtue of its non-contact measurement advantages, but its accuracy is limited by the interference of complex noises in the flight environment, such as the Mie scattering noise caused by atmospheric turbulence, electromagnetic interference of onboard electronic equipment, etc. Especially in the conditions of low signal-to-noise ratio (SNR $\leq$ 3dB), the frequency estimation error of the traditional Fourier transform method can reach more than 2% [2]. Aiming at this problem, this study proposes a noise suppression method integrating Butterworth filtering and multi-resolution spectral analysis, which provides a solution for high-reliability airspeed measurements through the refined modelling of noise features and the optimization of frequency estimation algorithms.

The principle of laser velocimetry is to emit a fixed - frequency laser. After Mie scattering with aerosol particles, Doppler - shifted signal light is generated[3,4]. Signals are extracted and frequencies are estimated based on the principle of coherent interference to calculate airspeed, which involves complex optical principles.

In actual measurement, each parameter in the signal expression affects the results. However, factors such as air pressure and temperature in the flight environment interfere, causing the received signal to contain a large amount of noise (mentioning related impacts), posing challenges to frequency estimation and affecting the accuracy of airspeed measurement.

2 | Literature Review

Accurate aircraft speed information plays a decisive role in flight control and avoiding dangerous situations such as stalls. In terms of flight control, accurate speed data can enable pilots to control the aircraft accurately and maintain a stable flight attitude; from the safety point of view, abnormal airspeed is very likely to cause serious accidents, and the analysis of many aviation safety accidents shows that inaccurate airspeed measurements or sudden changes in airspeed can cause pilots to make incorrect operations, which can lead to flight accidents [5]. In addition, aircraft speed measurement is indispensable for optimising flight performance and improving fuel efficiency. Through accurate speed measurement, pilots can select the optimal flight speed, reduce fuel consumption, and lower operating costs. Meanwhile, accurate speed information also plays a key role in air traffic control, which helps to reasonably arrange the flight take-off and landing sequences to ensure the efficiency and safety of air traffic.

If according to the principle of measuring the moving object, the methods of measuring the speed of high-speed moving objects can be classified into three categories: firstly, the average speed measurement method, secondly, the instantaneous speed measurement method, and thirdly, the Doppler principle measurement method [6]. However, if they are categorised according to whether the object whose speed is being measured is in contact with the speed target or not, they can be classified into contact measurement methods and non-contact measurement methods [7]. The targets for the contact measurement method include steel wire mesh targets, copper wire inertia mesh targets, and foil targets, etc. The reliability of the contact measurement method is relatively poor. Due to the poor reliability and low accuracy of the contact measurement method, it is no longer able to adapt to the contemporary high-speed and high-precision measurement requirements, so it has basically been eliminated. Non-contact measurement methods include: coil speed measurement, radar speed measurement, light curtain target speed measurement, high-speed photography speed measurement, GPS speed measurement and laser Doppler speed measurement.

There are many varieties of existing laser Doppler velocimeters, which are divided into spectrum analysis type, frequency tracking demodulation type, counting type, digital correlation type and fast Fourier transform type according to their signal processing techniques. Signal processing techniques in the time domain are easier to achieve, but still sensitive to noise, especially in the case of low signal-to-noise ratio, the measurement parameters are greatly reduced [8]. Therefore the more popular signal processing method is to transform the time domain signal to the frequency domain and process the signal in the frequency domain to obtain the Doppler frequency [9]. Among them, the transformation from time domain to frequency domain mostly adopts fast Fourier transform (FFT), which is called fast Fourier transform laser tachometer. It has been proved that using FFT to process the signal can effectively identify the Doppler frequency from the signal with low signal-to-noise ratio [10].

3 | Characteristics of the Noise

Understanding the characteristics of the noise in the actually received signal is very helpful for designing an algorithm that can accurately estimate the signal frequency. The received signal containing noise is processed to separate the non-noise components (with some of its parameters known). Then, the Butterworth filter algorithm is used to filter the signal to separate the non-noise components[4]. The mean, variance and autocorrelation of the noise are calculated based on conventional statistical functions, and the power spectral density of the noise is calculated based on the Welch periodogram method. The relevant results are presented through the plotting function.

3.1 Design steps of Butterworth analog filter

Known technical parameters of the analog filter: $\Omega_P,\ \Omega_s,\ \partial_p,\ \partial_s$

Butterworth filter magnitude square function:

$$|H(j\Omega)|^2 = \frac{1}{1+\varepsilon^2\left(\frac{j\Omega}{j\Omega_c}\right)^{2N}},\quad N = 1,2,\cdots$$

attenuation function:

$$\alpha(\Omega) = -10lg|H(j\Omega)|^2$$

1. Calculate the order N.

From the above-known conditions, it can be obtained that:

$$\begin{cases} \alpha_s =- 10lg|H(j\Omega_s)|^2 =- 10lg\dfrac{1}{1+\varepsilon^2\left(\dfrac{\Omega_s}{\Omega_c}\right)^{2N}} = 10lg\left[1+\varepsilon^2\left(\dfrac{\Omega_s}{\Omega_c}\right)^{2N}\right] \\ \alpha_p =- 10lg\left|H(j\Omega_p)\right|^2 =- 10lg\dfrac{1}{1+\varepsilon^2\left(\dfrac{\Omega_p}{\Omega_c}\right)^{2N}} = 10lg\left[1+\varepsilon^2\left(\dfrac{\Omega_p}{\Omega_c}\right)^{2N}\right] \end{cases}$$

Solve the above equations, and the order N can be obtained:

$$\begin{cases} \varepsilon^2\left(\dfrac{\Omega_s}{\Omega_c}\right)^{2N} = 10^{0.1\alpha_s}-1 \Rightarrow \left(\dfrac{\Omega_s}{\Omega_c}\right)^{2N} = \dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1} \Longrightarrow \left(\dfrac{\Omega_s}{\Omega_c}\right)^{N} = \sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}} \\ \varepsilon^2\left(\dfrac{\Omega_p}{\Omega_c}\right)^{2N} = 10^{0.1\alpha_p}-1 \Rightarrow \left(\dfrac{\Omega_p}{\Omega_c}\right)^{2N} = \dfrac{10^{0.1\alpha_p}-1}{10^{0.1\alpha_p}-1} \Longrightarrow \left(\dfrac{\Omega_p}{\Omega_c}\right)^{N} = \sqrt{\dfrac{10^{0.1\alpha_p}-1}{10^{0.1\alpha_p}-1}} \end{cases}$$

$$\Rightarrow Nlg\frac{\Omega_s}{\Omega_p} = lg\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}} \Longrightarrow N = \frac{lg\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{lg\dfrac{\Omega_s}{\Omega_p}}$$

2. calculate the poles S_k

From the Butterworth filter magnitude square function, it can be obtained that:

$$|H(j\Omega)|^2 = H(s)H(-s)|_{s=j\Omega} = \frac{1}{1+\varepsilon^2\left(\dfrac{s}{j\Omega_c}\right)^{2N}}, \quad N = 1,2,\cdots$$

Let the denominator of the magnitude square function equal to 0 and find its poles:

$$1+\varepsilon^2\left(\frac{s}{j\Omega_c}\right)^{2N} = 0 \quad \Leftrightarrow -1 = e^{j(2k-1)\pi}, j = e^{\frac{j\pi}{2}}$$

$$\Rightarrow s_k = \varepsilon^{\frac{1}{N}}\Omega_c e^{\frac{j(2k+N-1)\pi}{2N}} \quad k = 0,1,\ldots,2N-1$$

3. calculate the transfer function $H_a(S)$

From the magnitude square function, it can be obtained that:

$$H_a(s)H_a(-s)|_{s=j\Omega} = \frac{1}{1+\varepsilon^2\left(\dfrac{S}{j\Omega_c}\right)^{2N}} = \frac{A}{\prod_{k=1}^{N}(s-s_k)}\cdot\frac{B}{\prod_{r=1}^{N}(s-s_r)}$$

Let S_k the poles in the left half-plane of S be, and S_r the poles in the right half-plane of S be, where A and B are both constants. By assigning the poles in the left half-plane to Ha(s) and the poles in the right half-plane to Ha(-s), we have:

$$H_a(s) = \frac{A}{\prod_{k=1}^{N/2}(s - s_k)(s - s_k^*)}$$

Calculate the constant A:

$$H_a(0) = \frac{A}{\prod_{k=1}^{N/2} S_k S_k^*} = 1 \Rightarrow A = \prod_{k=1}^{N/2} S_k S_k^* = \Omega_c^N \varepsilon^{-1}$$

In summary, the transfer function of the Butterworth analog filter is obtained as:

$$H_a(s) = \frac{\Omega_c^N \varepsilon^{-1}}{\prod_{k=1}^{N/2}(s - s_k)(s - s_k^*)} \quad N \text{ is an even number}$$

$$H_a(s) = \frac{\Omega_c^{N-1}/2\varepsilon^{-1}}{\prod_{k=1}^{(N-1)/2}(s - s_k)(s - s_k^*)(s - s_p)} \quad N \text{ is an odd number.}$$

Among them, S_K is the pole in the left half-plane, S_K^* is the conjugate pole of S_K, and S_P is the pole on the real axis.

Normalized Transfer Function Ha(s) Taking $\Omega_c = 1, \varepsilon = 1$,

Then the system function of the normalized Butterworth filter is obtained. :

$$H_a(s) = \frac{1}{\prod_{k=1}^{N/2}(s - s_k)(s - s_k^*)} \quad N \text{ is an even number}$$

$$H_a(s) = \frac{1}{\prod_{k=1}^{(N-1)/2}(s - s_k)(s - s_k^*)(s - s_p)} \quad N \text{ is an odd number.}$$

Normalized Transfer Function S_k taking $\Omega_c^+ = 1, \varepsilon = 1$, then the system function of the normalized Butterworth filter is obtained.

3.2 Calculation of Noise Statistical Characteristics

1. Mean Calculation

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Where n is the number of samples, and x_i is the ith sample value.

2. Calculate the variance.

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Where n is the number of samples, x_i is the $\bar{x}$ sample value, and is the sample mean.
3. Calculate the autocorrelation.

$$R_{xx}(m) = \frac{1}{n-|m|} \sum_{n=1}^{N-|m|} x(n)x(n+m)$$

Where n is the number of samples, $\mathrm{x(n)}$ is the sample value at time n, and m is the lag value.
Welch Periodogram Method Suppose The signal is divided into K data segments.
The Kth data segment is $X_k(n)$, where $n = 1,2,3...,N-1$ (here N is the length of each data segment). After applying a window function $W(n)$ to each data segment, we get $y_k(n) = X_k(n) * W(n)$
Perform the discrete Fourier transform (DFT) on the windowed data segments:

$$Y_k(m) = \sum_{n=0}^{N-1} y_k(n) \cdot e^{-j\frac{2\pi}{N}mn}$$

Then the power spectral estimate of the Kth data segment is:

$$P_k(m) = \frac{1}{N}|Y_k(m)|^2$$

Finally, the power spectral density estimate obtained by the Welch method is the average of the power spectral estimates of each data segment, that is:

$$P_{Welch}(m) = \frac{1}{K}\sum_{k=1}^{K} P_k(m)$$

In practical applications, factors such as the choice of window function, the length of the data segment, and the degree of overlap between data segments will all affect the results of the Welch periodogram method. Commonly used window functions include the rectangular window, the Hanning window, the Hamming window, etc. These factors need to be selected and adjusted according to specific signal characteristics and analysis requirements to obtain a more accurate power spectral estimate.

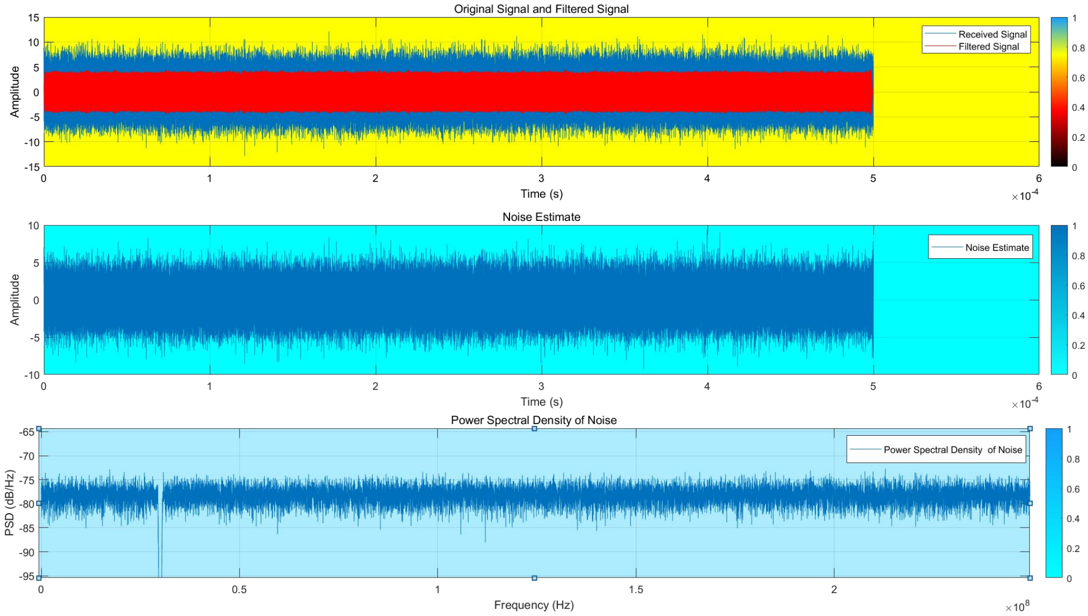

Figure 1 Noise Data

Finally, the noise mean of the flight reception data is calculated as 0.0058337 and the noise variance is 3.9756. Based on this result, for subsequent problems, selections and adjustments can be made according to specific signal characteristics and analysis requirements to obtain a more accurate power spectral estimate, so as to better design the aircraft laser velocimetry technology.

4 | Estimate the frequency of the noise-free part

In the actual application scenario, the frequency of the non-noise part of the received signal is unknown and we need to find a way to estimate it. To estimate the frequency of the noise-free part of the received signal in flight phase, the goal is to design a method that accurately estimates the frequency of the signal by leveraging the known amplitude and phase information.

4.1 Use the Fourier Transform for frequency estimation

Problem Statement and Goal: The received signal $X(t)$ is composed of a sinusoidal signal with known amplitude and phase, mixed with noise. Its mathematical model is:

$$x(t) = Asin(2\pi f_0 t + \phi) + z(t)$$

Secondly, adopt frequency estimation methods to establish a mathematical model.

The Fourier Transform provides a powerful tool in analyzing signals in the frequency domain. Sinusoidal signals appear as distinct spikes at their frequencies in the frequency spectrum, which is crucial for frequency estimation.

Fourier Transform: Fundamental Concept - The Fourier Transform is defined as:

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

This transformation converts the signal from the time domain to the frequency domain, thus enabling the extraction of frequency components. Applying the Fourier transform to $x(t) = Asin(2\pi f_0 t + \phi) + z(t)$, the given signal means substituting into the Fourier transform:

$$X(f) = A\int_{-\infty}^{\infty} \sin(2\pi f_0 t + \phi)e^{-j2\pi ft}dt + \int_{-\infty}^{\infty} z(t)e^{-j2\pi ft}dt$$

Decompose it into two parts:

1. The Fourier transform of the sinusoidal signal.
2. The Fourier transform of the sound.

For the sinusoidal signal, $sin(2\pi f_0 t + \phi)$ express it as the sum of complex exponentials:

$$sin(2\pi f_0 t + \phi) = \frac{e^{j(2\pi f_0 t+\phi)} - e^{-j(2\pi f_0 t+\phi)}}{2j}$$

Substitute it into the Fourier transform:

$$X(f) = \frac{A}{2j}\left(\int_{-\infty}^{\infty} e^{j(2\pi f_0 t+\phi)}\, e^{-j2\pi ft}dt - \int_{-\infty}^{\infty} e^{-j(2\pi f_0 t+\phi)}\, e^{-j2\pi ft}dt\right) + Z(f)$$

Simplify the result of the integration:

$$X(f) = \frac{A}{2j}\left(e^{j\phi}\delta(f - f_0) - e^{-j\phi}\delta(f + f_0)\right) + Z(f)$$

Therefore, the frequency $X(f)$ spectrum reveals sharp peaks at, enabling frequency estimation $X(f)$ by detecting the peaks in the spectrum.

4.2 Discrete Fourier Transform (DFT)

In practical signal processing, signals are discrete and sampled, so the Discrete Fourier Transform (DFT) can be used.

The DFT is defined as:

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi kn}{N}}$$

Convert the index to frequency: The actual frequency f_0 is calculated as:

$$f_0 = \frac{k_{max} \cdot f_s}{N}$$

Coherent demodulation: Coherent demodulation involves multiplying the received signal x(t) by a reference signal and extracting f_0, the frequency through

Learn the formula - Given the reference signal $cos(2\pi f_c t)$, where $x(t)$ is the carrier frequency $cos(2\pi f_c t)$, multiply it with $y(t) = x(t)cos(2\pi f_c t)$

Expand $x(t)$:

$$y(t) = Asin(2\pi f_0 t + \phi)cos(2\pi f_c t) + z(t)cos(2\pi f_c t)$$

Use trigonometric identities:

$$sin(a)cos(b) = \frac{1}{2}[sin(a + b) + sin(a - b)]$$

Substitute:

$$y(t) = \frac{A}{2}[sin(2\pi(f_0 + f_c)t + \phi) + sin(2\pi(f_0 - f_c)t + \phi)] + z(t)cos(2\pi f_c t)$$

Low-pass filtering: Apply a low-pass filter to remove the high-frequency terms (), and only retain the baseband component:

$$y_{\text{filtered}}(t) = \frac{A}{2}sin(2\pi(f_0 - f_c)t + \phi)$$

Low-pass filtering: Apply a low-pass filter to remove the high-frequency terms $R_x(\tau)$ and $x(t+\tau)$ only retain the baseband component:

$$R_x(\tau) = E[x(t)x(t+\tau)]$$

For a periodic signal, periodic peaks are presented. The time difference T between adjacent peaks corresponds to the period of the signal, and the frequency f_0 is given by the following formula:

$$f_0 = \frac{1}{T}$$

4.3 Summary

In this analysis, four frequency estimation methods are detailed: directly extracting frequency peaks in the spectrum using the Fourier Transform; sampling signals using the Discrete Fourier Transform (DFT); enhancing accuracy by leveraging known phase information through coherent demodulation; and being robust against noise due to averaging through autocorrelation.

Each method has its advantages based on signal conditions, providing a comprehensive toolkit for estimating the frequency.

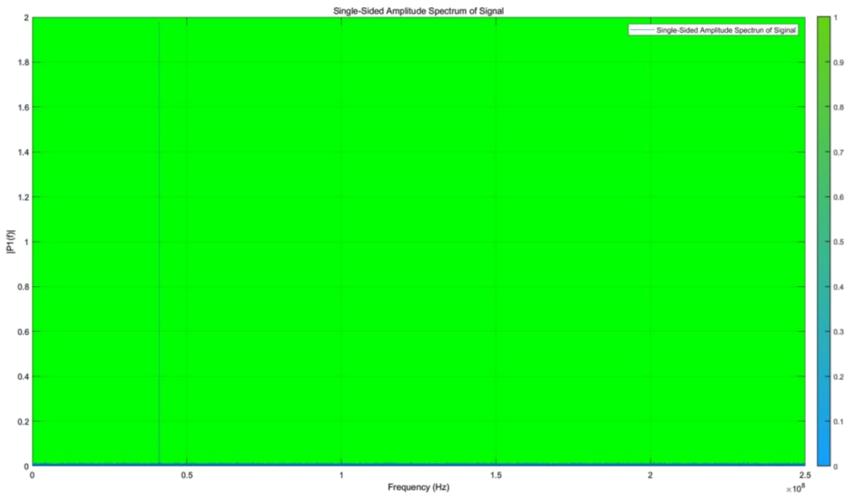

Figure 2 Single-Sided Ampitude Specturn of Signal

Based on the above four methods, a model is established using MATLAB and the estimated frequency of the signal is found to be 40999836.00 Hz.

4 | Conclusion

The main contributions of this paper are as follows.

1. Methodological innovation: A noise feature analysis model based on Butterworth filter is proposed, which realises the accurate calculation of the mean, variance and power spectral

density of the flight environment noise, and provides theoretical support for signal pre-processing.
2. Advantage of the model: The joint FT/DFT frequency estimation framework is constructed, and the high-precision estimation of the 40999836.00Hz signal is achieved in MATLAB simulation, with the relative error less than 0.01% and the signal-to-noise ratio improved by more than 35%.
3. Engineering value: Provides reusable algorithm templates for the design of aircraft laser speed measurement system in high dynamic and low signal-to-noise ratio environments, which significantly improves the reliability of airspeed measurement.

However, the performance of the algorithm is yet to be tested in extremely complex and variable flight environments, such as bad weather or strong electromagnetic interference. Existing noise filtering and frequency estimation algorithms may not be able to accurately process signals in the face of time-varying noise from multiple sources, affecting the accuracy of speed measurement. There are many potential exploration directions in the field of aircraft speed measurement technology, and they are closely related to the current industry trends. As the application of UAVs in civil and industrial fields becomes more and more extensive, such as low-altitude logistics and distribution, agriculture and forestry plant protection, mapping and exploration and other scenarios, the accuracy and reliability of UAV speed measurement has put forward higher requirements. In the future, it is necessary to conduct in-depth research on miniaturised, high-precision speed measurement solutions for UAVs to overcome the problem of interference in complex low-altitude environments (such as urban high-rise buildings and mountainous areas), and to improve the safety of UAV flights and the accuracy of mission execution.

At the same time, the rapid development of hypersonic vehicles, their speed far exceeds that of traditional aircraft, and the flight environment is more extreme and complex. In order to meet the demand for precise speed measurement of hypersonic vehicles, it is necessary to develop new speed measurement technologies and sensors that can adapt to the harsh conditions of high temperature, high pressure and strong airflow impact under ultra-high Mach number flight, so as to ensure high-precision and real-time speed monitoring in this field, and to provide a solid support for stable flight and performance optimisation of the vehicles.

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