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Reducing Costs and Improving Quality: Mathematical Modelling Strategies in Electronics Production

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abstract

In the production of electronic products, the testing process is crucial, but full-scale testing can bring about a conflict between cost and capacity. This study focuses on this and adopts a sampling testing method to balance the two relationships. The study first assumes that the defective rate conforms to the binomial distribution, approximates the normal distribution in the case of large samples, constructs the null hypothesis and alternative hypothesis, and determines the number of samples for sampling and testing at different confidence levels through statistical methods. At the same time, the testing cost, defective rate and market loss are considered comprehensively, relevant variables and constraints are constructed, and quality control decisions are formulated. The optimal decision under specific conditions is finally derived, which provides an effective quality control strategy for enterprises.

Keywords: Electronics Production; Sampling and Testing; Quality Control; Cost-benefit Analysis.

1 | Introduction

In the production of modern electronic products, testing is an essential part of the process, which will ensure that defective products will not flow out of the production line, into the hands of customers [1]. Therefore, any modern production company will not fail to tightly control the test to ensure that a significant proportion of the product performance indicators to meet the requirements [2]. Any omission of a defective product can lead to quality problems. However, a large number of comprehensive parametric tests consume considerable human and material resources. The survival and development of enterprises need to control costs and continuously improve production capacity, thus comprehensive testing and improving production capacity are bound to become a contradiction. In the field of production testing of electronic products, engineers and technicians have been committed to minimising testing or shortening the test cycle, so as to control costs to meet the needs of mass production [3].

2 | Literature Review

Since the beginning of international trade, the quality of products has been the focus of attention for both buyers and producers. Economic losses caused by quality are much higher than those caused by other factors. Quality inspection as a guarantee of export product quality is an indispensable means of international trade and is an extremely important part of the process. Quality inspection has a full number of tests and sampling tests two kinds, most countries use a more economical and practical sampling test method when purchasing [4].

Sample testing is a discipline with a hundred years of history. In the early 20th century, with the level of science and technology getting higher and higher, production efficiency continues to improve, the scale of production is getting bigger and bigger. During the First World War, due to the needs of the war, The production scale of military enterprises in the United States has increased sharply, and the output has increased greatly. The original 100% inspection of industrial products for the use of ex post facto checking method at this time exposed serious problems: (1) Due to the need for 100% inspection, making the inspection of the efficiency of the extremely low, and simply can not meet the needs of production and use. Under the pressure of production, enterprises have to increase a large number of inspectors. Some companies engaged in the inspection of the number of personnel and even more than the number of manufacturing personnel, even if this also produces products because it is too late to test the situation can not be shipped in a timely manner, resulting in customer complaints, or even cancellation of the order contract, and the serious consequences of the claim [5]. (2) As a result of the adoption of 100% inspection of the ex post facto checking method, no control of the production process, the inspection of a large number of rejected products caused the enterprise to lose a lot of money, and the loss can not be recovered [6]. (3) for destructive inspection can not use the full inspection, once the full inspection, all products will lose all or part of the function. In this case, many companies recognise that only now this full inspection to control product quality is certainly not feasible, must take other more advanced methods instead.

Sampling test standards as the embodiment of sampling test methods have developed rapidly in recent decades. The acceptable quality limit (AQL) as a quality indicator of a series of sampling inspection standards are widely used [7]. China and the International Organisation for Standardisation (ISO), the United States, Japan, Germany and other industrially developed countries, the current sampling inspection guidelines, acceptance and supervision of sampling methods, statistical theory and methodology, product-specific sampling requirements and other aspects of sampling inspection standards total more than 500. In China, so far, hundreds of sampling inspection standards have been issued, forming a more complete system. With China becoming the world's factory, sampling inspection of export products is one of the important methods to ensure quality and avoid claims [8].

3 | Number of Samples Taken for Testing

The supplier claims that a batch of spare parts will not exceed a certain nominal value. The enterprise is prepared to use sampling and testing to decide whether or not to accept the parts purchased from the supplier, and the testing costs will be borne by the enterprise. In this case, it is necessary to design a sampling programme with as few tests as possible [9].

The core of the programme is to determine statistically whether the defective rate of a product exceeds the nominal value declared by the supplier at a specific confidence level. We first assume that the defective rate conforms to the binomial distribution $X \sim b(n, p)$, and then approximate the statistic to a normal distribution by using a large sample test with a large sample size. By constructing the null hypothesis (the defective rate is equal to the nominal value) and the alternative hypothesis (the defective rate exceeds the nominal value), we can make the decision of whether to reject or accept the spare parts accordingly. This is done as follows.

Step 1. Building assumptions

Assumption 0: $p \leq p_0$, indicating that the true defect rate is not higher than the nominal value

Alternative assumption: $p > p_0$, indicating that the true defect rate is greater than the nominal value

For the above two hypotheses, the confidence levels of the objectives are 95% and 90% respectively, and within the tolerance of error, we consider the smallest possible sample size to judge whether the defective rate of spare parts is over or under the nominal amount, so as to reject or accept this batch of spare parts.

Step 2. Scenario analysis

Scenario 1: Rejection of spare parts with defective rates exceeding nominal values at a confidence level of 95%

In this case, we can set the criteria for rejection through hypothesis testing, rejecting samples when the rate of defective products detected by the sample is significantly higher than the nominal value within our error allowance.

Scenario 2: Receiving spare parts with lower than nominal reject rates at a confidence level of 90%

In this case, we can set the criteria for acceptance through hypothesis testing, so we have to check that the defective rate of the sample is within a reasonable range, which makes it possible to accept the batch of spare parts.

Step 3. Statistical modelling

We test the sample size based on binomial distribution and normal distribution approximation, the number of defective products in general obeys the binomial distribution, taking into account the detection value and the real defective rate of error, as well as the sample size is large, so the samples are made for the normal distribution approximation to deal with it, so as to further consider the test of the sample.

We consider the critical case in which the topic's requirements for confidence are met, within the limits allowed by the error. Based on the large sample test (using scenario 1 as an example), we can obtain the critical case:

$$u_{0.95} = \frac{\sqrt{n}(p_1 - p_0)}{\sqrt{p_1(1-p_1)}}$$

We multiply and divide $\sqrt{p_0 * (1-p_0)}$ simultaneously on the numerator, taking $p_1 - p_0$ as the mean error E_1 and $\frac{\sqrt{p_1(1-p_1)}}{\sqrt{p_0(1-p_0)}}$ as the bias error E_2 . Our objective function is

$$n^* = \max\{n \text{ of the solution under the allowable range of error}\}$$

In the solution process, we introduce an error variable with the goal of calculating the minimum sample size required to optimise the detection cost and improve the decision-making efficiency while ensuring that the error is within an acceptable range.

Step 4. Example of sample size calculation

Assuming our allowable errors $E_1 \in (-0.02, 0.02)$, $E_2 \in (0.9, 1.1)$, we can calculate the required sample size within the error allowance, i.e., our objective function is:

$$n^* = \min\{\min\{n|\text{within the allowable range of error}\} \text{ meet the reliability requirements}\}.$$

95% Sample size at confidence level:

$$n^* = (\frac{u_{0.95} * E_2}{E_1})^2 * p_0 * (1-p_0) = 1045.846$$

90% Sample size at confidence level:

$$n^* = (\frac{u_{0.90} * E_2}{E_1})^2 * p_0 * (1-p_0) = 732.24$$

Step 5. Final decision-making

With the confidence level of 95% , we need to test 1046 spare parts, and if the defective rate is more than 10% , we will reject the spare parts.

With the confidence level of 90% , we need to test 733 spare parts, and if the defective rate is more than 10% , we will reject the spare parts.

4 | Sample and Test decisions

We make key quality control decisions in real-world scenarios for an organisation's production of electronic products, including parts inspection, finished product inspection, non-conforming goods handling and customer returns management, as shown in Figure 1 below. We use a cost-benefit analysis to weigh inspection costs, defect rates, and market losses to determine the optimal strategy. The ultimate goal is to ensure product quality and improve customer satisfaction while controlling costs and maintaining corporate reputation [10].

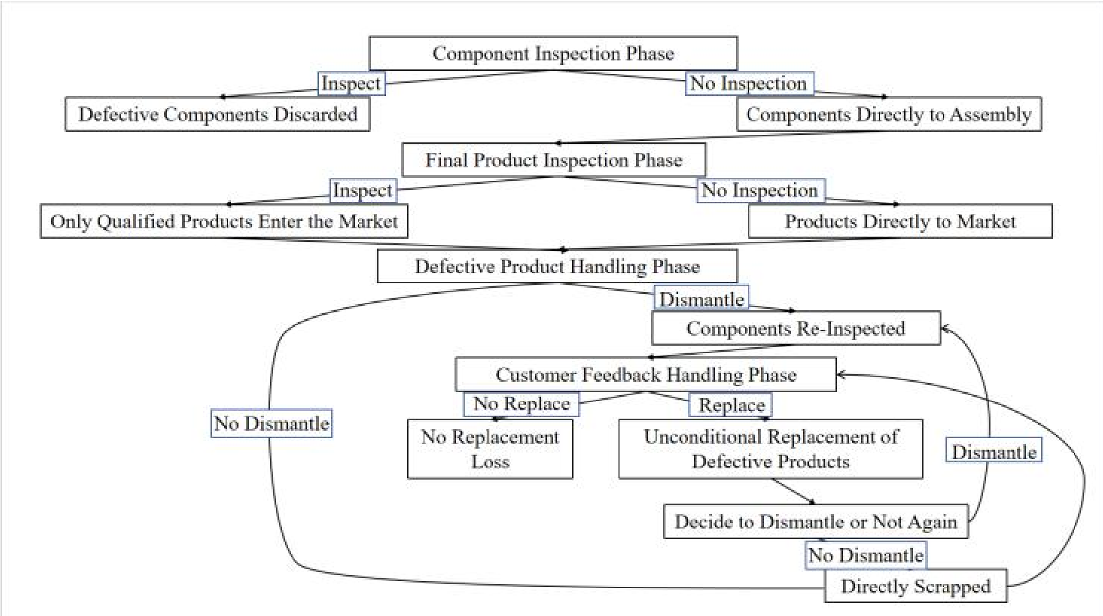

Figure 1 Diagram of specific decision points in the production process

The decision is centred on maximising profit by planning the various stages of the test in different scenarios, taking into account the company's reputation. Therefore, we set a constraint: the exchange rate $e \leq 5\%$. We form a decision set through the inspection decisions for spare parts 1 and 2 and the finished product inspection and dismantling decisions

(x_1, x_2, x_3, x_4) , The variables are x_1, x_2, x_4

(1) Constructing variables and constraints

Define the variables $\{x_1, x_2, x_3, x_4\} \in [0, 1]^4$, x_1, x_2 to denote the rate of whether or not Parts 1 and 2 are tested, x_3 to denote the rate of dismantling of finished products, and x_4 to denote the rate of testing of finished products, respectively. Based on the previous assumption "Considering the reputation of the enterprise in relation to the exchange rate, it is agreed that the exchange rate will not exceed 5% ". e represents the switching rate of the product, and the constraint condition can be obtained: $e \in [0, 0.05]$.

At the same time, we discuss the disassembly problem separately and compare it with the case without disassembly to obtain the optimal profit margin for each preliminary scenario p. Further, by comparing different profit margins for the same scenario, we determine the optimal decision and profit margin for each scenario. We analysed the inspection decisions for spare parts 1 and 2, as well as the inspection of the finished product and the dismantling option for non-conforming products to form multiple combinations of decision variables. By comparing costs and profits in the dismantling versus no dismantling scenarios, we evaluate the optimal profit margins for each initial option.

(2) Calculation of costs incurred at each stage

Considering that the costs of Parts 1 and 2 are determined by x_1 and x_2 respectively, the costs determined by Parts 1 and 2 are expressed in terms of $\{x_1, x_2\}$ to, and the relevant formulas are as follows:

$$Tc_1 = 4(1 - x_1) + \frac{(4 + c_1)x_1}{(1 - r_1)}$$

$$Tc_2 = 18(1 - x_2) + \frac{(18 + c_2)x_2}{(1 - r_2)}$$

Considering the cost of the finished product without testing only the cost of assembling spare parts under the above conditions, the cost formula for the finished product without testing is as follows:

$$Tc_3 = Tc_1 + Tc_2 + 6$$

Considering the cost of testing to get the finished product, it is necessary to consider the cost of inspection in addition to the cost of assembling spare parts, so the cost of testing to get the finished product is calculated as follows:

$$Tc_4 = Tc_1 + Tc_2 + 6 + c_3$$

In order to calculate whether the finished product is tested or not, we first consider the probability that both spare part 1 and spare part 2 pass the test, which is related to both x_1 and x_2. The formula for calculating the probability that both spare part 1 and spare part 2 pass the test is as follows:

$$\pi = [x_1 + (1 - r_1)(1 - x_1)][x_2 + (1 - r_2)(1 - x_2)]$$

Consider the incremental cost of switching, which is related to our assumptions, and the formula for the incremental cost of switching a product is as follows:

$$Tc_5 = [\frac{(4 + c_1)}{(1 - r_1)} + \frac{(18 + c_2)}{(1 - r_2)}] + s_1 + (c_3 + s_2 + 6)(1 - r_3)$$

In order to calculate the inspection rate of the finished product, we first consider the rate of sold products inspected. The formula for calculating the rate of sold products inspected is as follows:

$$n = \frac{1 - e}{1 - \pi(1 - r_3)}$$

In order to calculate the cost incurred for the inspection of finished goods, the inspection rate of finished goods is considered and the formula for the rate of finished goods to be inspected is as follows:

$$x_4 = \frac{\frac{n}{(1 - r_3)\pi}}{(1 - n) + \frac{n}{(1 - r_3)\pi}}$$

The total cost is the sum of the costs and in the above case we can derive the formula for the total cost as follows:

$$TC = x_4 * Tc_4 + \frac{(1 - x_4)Tc_3}{1 - r_3} + Tc_5 * e$$

(3) Calculation of preliminary profit margin

The above describes the calculation of the total cost, the following considers the profit, the profit is determined by the exchange rate, from the profit minus the cost to get the profit, the profit calculation formula is as follows:

$$q_1 = (1 - e)q - Tc$$

Considering that the profit margin is determined by profit and cost, we can get the formula for calculating the profit margin as shown below:

$$p_1 = \frac{q_1}{Tc}$$

(4) Consideration of new profit margins after dismantling

Consider the defective rate of the disassembled spare part 1, because we have already got the conclusion that the finished product is defective, we use the Bayesian formula to calculate the new defective rate of the spare part 1, and the new defective rate of the spare part 1 is calculated by the following formula:

$$r_1{}' = 1 - \frac{(1 - r_1)[r_3 + (1 - r_3)r_2]}{1 - (1 - r_3)(1 - r_2)(1 - r_1)}$$

Considering the defective rate of dismantled spare part 2, the formula for the new defective rate of spare part 2 obtained above is as follows.

$$r_2{}' = 1 - \frac{(1 - r_2)[r_3 + (1 - r_3) * r_1]}{1 - (1 - r_3)(1 - r_2)(1 - r_1)}$$

Considering the incremental cost of dismantling, which is a fixed parameter, the formula for calculating the incremental cost of dismantling is as follows:

$$Tc_6 = (c_1 + c_2 + s_2)\frac{1 - \pi(1 - r_3)}{\pi x_4(1 - r_3) + 1 - x_4}$$

The formula for calculating the new profit, taking into account the new costs and new profits generated by dismantling, is as follows:

$$q_2 = [(4 + c_1)(\frac{1 - r_1{}'}{1 - r_1}) + (18 + c_2)(\frac{1 - r_2{}'}{1 - r_2})]\frac{1 - \pi(1 - r_3)}{\pi x_4(1 - r_3) + 1 - x_4} - Tc_6$$

After the new profit margin after disaggregation, in summary, the formula for the new profit margin can be obtained as follows

$$p_2 = \frac{q_1 + q_2}{Tc + Tc_6}$$

(5) Derive the optimal decision

In each case, after considering the initial four scenarios, get p_1, p_2, to take the maximum value of the case to decide whether the solution is a disassembled decision, and then compare the four scenarios to get the maximum profitability, to find the corresponding x_1, x_2, to get the final solution. First of all, through the constraint $e \leq 0.5$, get the range of x_4, and then get the maximum profit margin p_1, p_2. Then get the maximum profit margin for each scenario, i.e.

$$p = max\{p_1, p_2\}$$

For each case, we arrive at the optimal decision by comparing the profitability p of each scenario.

(6) Optimisation of procedures and algorithms

Algorithmically, we consider annealing and greedy algorithms. This combined analysis helps us to find an optimal testing strategy that balances cost, risk and company reputation. We make 32 decisions for each case and the optimal strategy for each case is as follows:

Table 1 Optimal decision-making for various scenarios

	x_1	x_2	x_3	x_4	e	p
Scenario 1	0.00 %	0.00 %	100.00 %	100.00 %	0.00 %	55.50 %
Scenario 2	0.00 %	0.00 %	100.00 %	100.00 %	0.00 %	42.10%
Scenario 3	0.00 %	0.00 %	100.00 %	100.00 %	0.00 %	55.50 %
Scenario 4	0.00 %	0.00 %	100.00 %	100.00 %	0.00 %	45.20 %
Scenario 5	0.00 %	0.00 %	100.00 %	100.00 %	0.00 %	65.10%
Situation 6	0.00 %	0.00 %	100.00 %	100.00 %	0.00 %	54.90 %

The optimal decision in all cases is $\{x_1, x_2, x_3, x_4\} = \{0, 0, 1, 1\}$, i.e., the maximum profitability is obtained when neither spare part 1 nor spare part 2 is tested, and the finished product is tested and the failed products are dismantled.

5 | Conclusion

Focusing on the production testing of electronic products, this paper achieves key results in the determination of sample size for sampling and testing and the formulation of quality control decisions. By constructing a statistical model based on binomial distribution and normal distribution approximation, the required sample size for sampling and testing is calculated at different confidence levels, which provides enterprises with a clear basis for sampling and testing. Meanwhile, based on cost-benefit analysis, relevant variables and constraints are constructed to calculate the costs and profit margins at each stage, which in turn leads to the optimal decision-making for quality control in the production of electronic products, effectively balancing the testing cost, defective rate and market loss.

However, in the sampling and testing sample size model, it is assumed that the defective rate conforms to the binomial distribution and normal distribution approximation treatment, which may deviate from the real situation. In addition, in the quality control decision-making part, although several cost factors and constraints are considered, some factors that are difficult to quantify, such as changes in market dynamics and customers' psychological expectations, are not adequately considered, which may lead to a certain degree of impact on the accuracy of decision-making. Future research can optimise the sampling and testing model, consider introducing a distribution model that is more in line with the actual production situation, or adopt methods such as machine learning to mine data features to improve the accuracy of sample size calculation.

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